A11101 294945

NAT'L INST OF STANDARDS & TECH R.I.C.
A11101294945
/Bulletin of the Bureau of Standards
QC1 .U5 V7;1911 C.2 NBS-PUB-C 1905

DEPARTMENT OF COMMERCE AND LABOR

BULLETIN

OF THE

BUREAU OF STANDARDS

VOLUME 7

1911

DEPARTMENT OF COMMERCE AND LABOR

BULLETIN

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

VOLUME 7

1911

WASHINGTON
GOVERNMENT PRINTING OFFICE
1911

THE CAPACITY AND PHASE DIFFERENCE OF PARAFFINED PAPER CONDENSERS AS FUNCTIONS OF TEMPERATURE AND FREQUENCY

By Frederick W. Grover

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INTRODUCTION

The phenomenon of residual charges has long been observed and studied. The subject has been approached both by direct measurement of the quantity of electricity which appears, with the end in view of determining the laws governing its formation, and indirectly by measurement of the energy loss in dielectrics which exhibit this anomalous behavior.

To account for these evidences of a departure from the law of an ideal dielectric (according to which the charge at every moment is proportional to the instantaneous value of the impressed voltage) a number of theories of quite varied nature have been proposed, the most important of which will be considered below. Some of these are in qualitative agreement with the observed facts, but are at variance as regards the magnitude of the deviations from a perfect dielectric, while others give equations too complicated to admit of quantitative verification. Further, the great mass of the existing data refers to measurements with a steady emf, and under such a variety of conditions as regards temperature, length of charge, length of discharge, and previous treatment, as to render the task of deciding for or against any given theory well-nigh impossible.

With the employment of alternating current in the measurements a great simplification is gained, and at the same time a high degree of accuracy is attained. For, the condenser being regularly and rapidly carried through a definite cycle of charging and discharging, almost instantly reaches a condition of equilibrium, a condition not readily attained in measurements with a steady emf.

In view of these considerations, and following the development at the Bureau of Standards, of accurate and rapid methods for determining the loss of energy in a condenser, simultaneously with the measurement of the capacity, it was suggested to the author by Prof. Rosa, that an investigation of the change of capacity and absorption of condensers in which these changes are relatively large (for example, those employing paraffined paper as a dielectric) might throw some light on the points at issue.

As originally projected, the work was to include, first of all, an examination of a number of commercial condensers already at hand, to be followed by the construction and measurement of condensers prepared under definite specifications, to test the effect of variations in the method of construction on the behavior of the completed condenser.

Only the first part of this plan has been carried out. In October to December, 1906, a series of measurements at two frequencies,

33 and 100 cycles per second, was made on 12 different condensers, and at temperatures ranging from 10° to 35° in steps of 5°. This work was extended to other frequencies in July, 1907, the same condensers, together with one added to the previous list, being measured at 100, 300, 600, and 900 to 1000 cycles, the measurements at 100 cycles having been repeated to connect with the previous work. Owing to lack of time, only three temperatures, 15°, 25°, and 35° could be included in the last set. The work was interrupted at this point by the absence of the author from the Bureau, and following his return, other more pressing work has intervened to prevent the completion of the original plan. It has seemed, therefore, best to present without further delay, the results of the work already completed, together with such points of theoretical interest as have presented themselves in the discussion of the observations.

The first part of the present paper is accordingly devoted to a description of the method of measurement and the results of the observations, while the latter part deals with the theories of absorption, and a comparison of the observed facts with the theoretical requirements.

I. MEASUREMENTS

Only in a condenser having a perfect dielectric, i. e., one which is free from absorption and whose leads and plates have a negligibly small resistance, does the difference of phase between the current in the condenser and the sinusoidal emf impressed upon it equal the value of 90° required by theory. In all condensers which show the phenomenon of the formation of residual charge, the angle of advance of the current with respect to the impressed emf falls short of 90° by an angle ϕ whose sine is a measure of the energy dissipated in the condenser. This angle, which is usually small, is important for another reason. It is found to be, to a large degree, a measure of the relative magnitude of the residual charges of the condenser. In what follows, the angle ϕ will be designated simply as the *phase difference* of the condenser.

For the measurement of the phase difference, advantage is taken of the fact that, to obtain a balance in a Wheatstone bridge in which two of the arms contain two condensers whose capacities

are being compared, two conditions must simultaneously be fulfilled, (*a*) the potential differences over the two condenser arms must be equal, and (*b*) these electromotive forces must have the same phase. The latter condition involves directly the difference in magnitude of the phase differences of the two condensers.

This matter has been considered in considerable detail in a previous paper,[1] where a number of alternating current bridge methods for obtaining at one and the same time the capacity and phase difference of a condenser, with reference to those of a standard were discussed, both from a theoretical and a practical standpoint. It will only be necessary here to recall such of the conclusions and formulas of the former paper as are applicable to the present measurements.

1. METHODS

In the earlier part of the work (frequencies 33 and 100) the Series Inductance Method (loc. cit., p. 390) was employed. In this

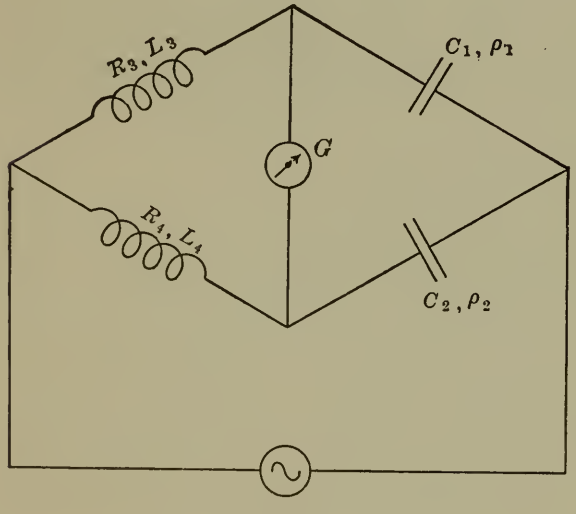

Fig. 1

NOMENCLATURE

R_3 and R_4 are resistances associated with the inductances L_3 and L_4 of which L_3 is variable. ρ_1 and ρ_2 are fictitious resistances in series with the condensers C_1 and C_2 which would produce the same shift of phase as the absorptions of the condensers.

method, the difference in the absorptions of the two condensers is balanced by a variable inductance in one or both of the ratio arms. In accordance with previous experience, the condenser whose constants were to be measured, was compared with the standard of reference—a condenser of known capacity and phase difference—by substitution, the unknown and the standard being successively balanced against an auxiliary condenser C_2 (Fig. 1) whose

[1] This Bulletin, **3**, p. 371; 1907.

value does not need to be known, but whose capacity should be of the same order of magnitude as those of the other two condensers. This procedure has the advantage of eliminating capacity effects between the various parts of the bridge and the earth and between one another, which effects can become a fruitful source of error at the higher frequencies.

The conditions for balance of the bridge shown in Fig. 1, when the substitution method is used, are

$$\frac{C_1'}{C_1''} = \frac{R_3''}{R_3'}\left[1 + \tan(\varphi_1' - \varphi_1'')\left\{\tan\varphi_2 - \frac{pL_4}{R_4}\right\}\right] \quad (1)$$

$$\tan(\varphi_1' - \varphi_2'') = \frac{pL_3'}{R_3'} - \frac{pL_3''}{R_3''} \quad (2)$$

The correction term in the formula for the ratio of the capacities can be made very small, even in the case of condensers with large absorption, by so choosing L_4 and R_4 that the angle of lag in that arm of the bridge approximately equals the phase difference of the auxiliary condenser. This is, however, not important, since in the case of condensers whose absorption is sufficiently large to give to $\tan(\varphi_1' - \varphi_1'')$ an appreciable value, the capacity is always observed to be so susceptible to minute temperature changes and slight variations in the frequency as to obscure the small effect of this correction term.

For example, with a paper condenser of moderately large absorption, and with the constants of the bridge used as follows:

$p = 600$, $L_2 = 10$ millihenrys, $R_4 = 1200$, $\varphi_1' - \varphi_1'' = 30'$, $\varphi_2 = 4'$

the correction to the capacity is only

$$\tan 30'\left\{\tan 4' - \frac{6}{1200}\right\} = -0.000029$$

which is practically negligible with such a condenser.

The later work was carried out almost exclusively by means of the Series Resistance Method (loc. cit., pp. 380, 387), in which the difference in absorption is compensated by inserting resistance in series with one or both of the condensers. This arrangement was

used as a substitution method as in the case of the Series Inductance Method.

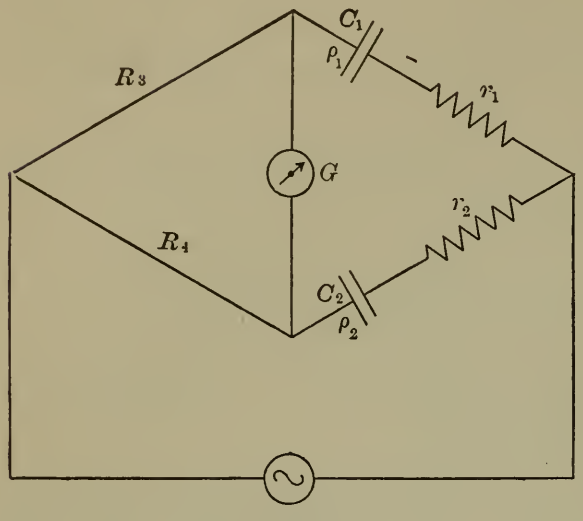

Fig. 2

NOMENCLATURE

R_3 and R_4 are the resistances of the ratio arms. C_1 and C_2 are condensers whose fictitious resistances are ρ_1 and ρ_2, and r_1, r_2 are resistances used in balancing the differences in the phase differences of the condensers. The standard and the unknown are successively inserted at C_1 and the values of R_3 and r_1 for balance are respectively R_3'', r_1'' and R_3', r_1'.

The conditions for balance are then very approximately

$$\frac{C_1'}{C_1''} = \frac{R_3''}{R_3'} \tag{3}$$

$$\tan(\varphi_1' - \varphi_1'') = pC_1''\left(r_1'' - \frac{R_3''}{R_3'}r_1'\right) \tag{4}$$

This method has the advantages over the preceding for work with condensers of relatively large phase difference, that it is much easier to vary the series resistance over a large range (as is necessary when testing condensers whose phase differences cover a large range of values) than it is to vary the inductance L_3. For instance, with the condensers here examined it was found to be necessary to obtain values of the inductance variable in rather small steps between a few millihenrys up to about 1 henry in the case of the worst condenser. With the fixed values of inductance usually at hand it becomes a tedious process to find the combination of fixed values which, taken together the variable inductance used, will give a balance of the bridge within the range of the variable.

Some difficulty was experienced in the work at the higher frequencies in obtaining trustworthy values of the phase differences. This was found to be due to the fact that the ratio coils were mounted in metal cases. Arranged thus, the capacity of

these coils with respect to their surroundings is so large, that at these high frequencies, even the small differences in the potential of the galvanometer terminals necessitated by the small difference of resistance in the ratio arm when passing from the standard to the unknown are sufficient to materially change the phase of the current in the arms of the bridge, and thus vitiate the value determined for the phase difference. Evidently this effect, which is negligible at the lower frequencies, is aggravated by any cause which gives the resistance coils a larger capacity, and its magnitude, other things being equal, will vary as the absolute values of the ratio arms are changed. Further, the fact that these paper condensers had capacities varying more from the values of the standards of comparison than is usual in precision work, tends to aggravate this effect.

It was found, however, that by using unmounted ratio coils of values not greater than about 500 to 1000 ohms, only the relatively small variable part of the ratio arm being included in a metal case, the values of the phase difference determined became sensibly independent of the magnitude of the ratio arm.

The resistances r, used in the condenser arms, were usually small, and were taken from dial boxes, a slide wire in series with the latter enabling the changes in the resistances of these arms to be measured to 0.001 ohm.

The effect of small resistances in the leads becomes very important in the measurement of the phase differences of condensers of 1 microfarad and greater at the higher frequencies used. For example, a resistance of only 0.02 ohm in the leads of a 1-microfarad condenser at 1000 cycles per second, increases the measured phase difference by 26″. The correction is proportional to the capacity and the frequency. This resistance was kept as small as possible by the use of stout wires soldered where practicable, and the actual resistance external to the condenser terminals was determined and corrected for in every case. At the lower frequencies some of the measurements were made using a two-way key to facilitate changing from the unknown to the standard. This was, however, abandoned in all work above 100 cycles, because of the chance of uncertain contacts in the condenser arm, which might

prove to be the source of an appreciable error in the determination of the phase difference.

Measurements were always made in pairs, using two different values of the ratio arms, both with the standard and the unknown. This procedure is well worth the additional time required, even when a large number of condensers are to be compared, because of the check thereby afforded, both on the measurements and the calculations. The latter are by no means complicated, and consume no undue amount of time. When, however, as in the present instance, many measurements are to be made, it would be a saving of time and would facilitate the detection of errors during the course of the observations, to use the arrangement devised by Curtis [2] (since the completion of this work) which allows the capacity of the unknown condenser to be read directly from the setting of the bridge.

2. APPARATUS

The current for the comparisons at 33 and 100 cycles was taken from a ½-kw motor-generator set, driven from a storage battery, the variations of the speed desired being obtained by a variable rheostat in the armature circuit of the motor. The higher frequency currents (300, 600, 900–1000 cycles) were supplied by three generators of a series designed to give frequencies corresponding to the fundamental and the odd harmonics (including the fifteenth) of 60 cycles. This set of alternators was specially designed for the Bureau of Standards for researches requiring a wide possibility of variation of wave forms of known composition.

Vibration galvanometers were employed to indicate when the bridge was balanced. Since an instrument of this type is highly sensitive only for a frequency corresponding to the natural period of torsional vibration of the suspended system, and responds only in a very small degree to currents of other frequencies, the emf does not have to be closely a sine wave. The galvanometer if tuned to the frequency of the fundamental, indicates when the bridge is balanced for the latter, although this condition does not hold for the other components.

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[2] This Bulletin, **6**, p. 439; 1910.

The importance of this property (which is not shared by the telephone) in measurements like the present, can not well be overestimated. In the case of a condenser circuit, all existing distortion of the emf wave is magnified in the current wave by the capacity. Since, now, unless the phase difference of both condensers is zero the conditions for balance of the bridge are different for each harmonic, any instrument like the telephone must always show the presence of current of some frequency, however the arms of the bridge are varied and with condensers of large absorption the minimum current will be very appreciable. The position corresponding to the minimum of the current will not, in general coincide with the balance point of the fundamental, and the sensibility of the setting is reduced by the presence of the residual of current.[3] These effects (small with condensers of small and nearly equal phase differences) are so great with condensers of large absorption, as to render the use of the telephone as indicating instrument unsatisfactory, if not impossible. In the present instance, at the lowest frequency, the difference in the balance point for the fundamental and that of the harmonics was in a few cases so great, as actually to produce appreciable forced vibrations of the galvanometer corresponding to the residual currents of harmonic frequencies. In such a case inductance may be used in the main circuit to annul the effects of the harmonics, or capacity may be introduced to produce more or less complete resonance of the circuit for the fundamental. With such poor condensers, however, this is hardly worth while, since it will be found that the balance is so sensitive to minute changes of the frequency, and the slight heating of the condenser by the *measuring current* causes such a notable drift of the setting for balance, as to enable snap readings only to be obtained.

For the measurements at 33 and 100 cycles two galvanometers of the Rubens type [4] were used; for the higher frequencies, instruments of the Wien type.[5] The latter form of vibration galvanometer is considerably the more sensitive, but suffers from the disadvantage that its resonance peak is inconveniently sharp, and its

[3] This Bulletin, **3**, p. 376; 1907.
[4] Wied. Ann., **56**, p. 27; 1895.
[5] Ann. der Phys., **4**, p. 425; 1901.

damping relatively poor. The Rubens galvanometer is free from these difficulties, and is very satisfactory for all work which does not demand unusual sensibility, but so far as is known to the author it is not made for frequencies higher than about 200 cycles per second. Recent experience indicates that the Campbell form [6] of vibration galvanometer combines with a greater sensibility a larger range of attainable frequencies than the Rubens.

No attempt was made to adjust the resonance frequencies of the Wien galvanometer closely to the exact values desired. It is a tedious operation to adjust the system of a galvanometer of this type to resonance with any prescribed frequency, but it is fairly easy to approximate the desired frequency in a very few trials, and to vary the speed of the machine until the current is in resonance with the period of the galvanometer. It is a simple matter to correct the observed results for the small difference between the actual and the desired frequency.

3. CONTROL AND MEASUREMENT OF THE FREQUENCY

Although a precise knowledge of the value of the frequency is not requisite in these measurements, means must be provided by which the resonance frequency may be located and thereafter reproduced at will. This is especially important when a setting is to be made on a condenser with an appreciable phase difference. If the speed of the machine fluctuates, the balance of the bridge will drift in a manner very annoying if many settings are to be made. Further, with the Wien galvanometer, the resonance peak is so sharp that a variation of the frequency by only one-tenth of a per cent from the resonance value reduces the deflection (other things being equal) to less than one-half of its value, so that it is often easy to obtain a spurious balance due to the reduced sensibility of the galvanometer. In any case it becomes very arduous to balance the bridge with a continually varying sensibility.

With the lower frequencies the speed of the alternator was adjusted by an auxiliary Maxwell bridge, the condenser being charged and discharged by a rotating commutator direct connected

[6] Phil. Mag. [6], 14, p. 794; 1907.

to the shaft of the alternator.[7] This method not only allows the resonance point to be very exactly identified and reproduced, but with a calibrated condenser the corresponding absolute value of the frequency may be determined with great precision.[8]

At the beginning of the day's work the vibration galvanometer is connected to a simple Wheatstone's bridge (or any other bridge whose balance does not depend on the frequency of the current). Having nearly balanced the bridge, so that the galvanometer shows only a small deflection, the speed of the generator is varied until the resonance frequency is attained, as evidenced by a sharply defined maximum deflection of the galvanometer.

This speed being maintained constant by one observer, a second observer balances the Maxwell bridge by adjusting the resistance in one of the arms. This having been accomplished, the resonance frequency can thereafter be reproduced and maintained constant by one observer at the Maxwell bridge whenever the alternating current bridge is to be balanced by the second observer.

This method was not found practicable in the case of the measurements at the higher frequencies, where the alternators were in a separate building from the bridge. Well-insulated leads were carried between the two buildings, but, owing perhaps to variations in the capacity between the leads and the earth, the galvanometer deflection varied in an erratic manner, and the attempt was abandoned.

The following method, suggested by Prof. Rosa was found very effective, and has the advantage that it is independent of any adjustment of brushes, an advantage not to be overlooked when the alternator is in another room from the observer.

This method depends on the use of a double electrodynamometer having two movable coils fixed to the same axis, each being placed in the field of a pair of fixed coils. In the instrument used in these measurements a taut suspension of phosphor bronze strip stretched between spiral springs at both top and bottom and provided with a damping vane insured a dead-beat movement, one of the requisites for the success of the method. The actual instrument was

[7] This Bulletin **1**, p. 153; 1904. [8] This Bulletin, **1**, p. 181; 1904.

inexpensive and of only moderate sensibility, but gave a very satisfactory performance.

The whole or a portion of the supply voltage is impressed on two circuits in parallel (Fig. 3), in one of which is connected in series a condenser, the two movable coils m_1 and m_2 and one pair f_2 of the fixed coils of the double electrodynamometer, and in the other a large inductance L and the remaining fixed coils f_1. Fig. 4 shows the relative phases of the currents in the dynamometer. The torque between the fixed coils f_1 and the movable coils m_1 is proportional to $i_1\, i_2 \cos \phi$ and that between the fixed coils f_2 and the movable coils m_2 is proportional to i_1^2, where i_1 and i_2 are, respectively, the currents in f_1 and f_2. But $i_1 \propto pC$ and $i_2 \propto \frac{I}{pL}$ provided the reactances of the coils of the dynamometer are small in comparison with those of the condenser and the inductance coil. The torque t_1 is therefore dependent to only a slight degree on the frequency, while the torque t_2 varies nearly as the square of the frequency. The connections being so made that the two torques t_1 and t_2 are opposed to one another, it is possible by adjusting the capacity or inductance, or both, to bring the deflection of the dynamometer to zero. This balance having been made for a given frequency, it is evident that any change of the frequency will disturb the balance and, what is more, the deflection occurring with decreasing frequency will be in the opposite direction to that with increasing frequency.

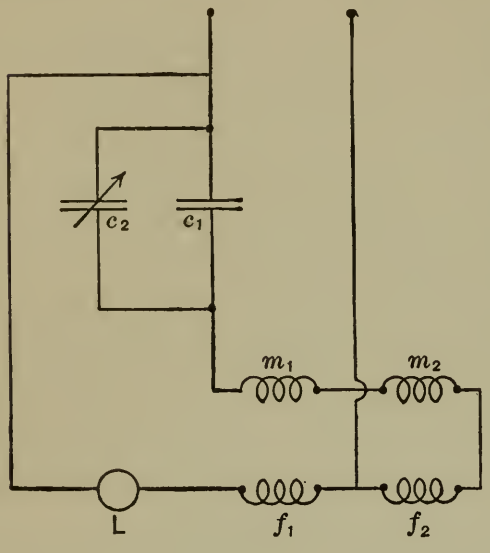

Fig. 3

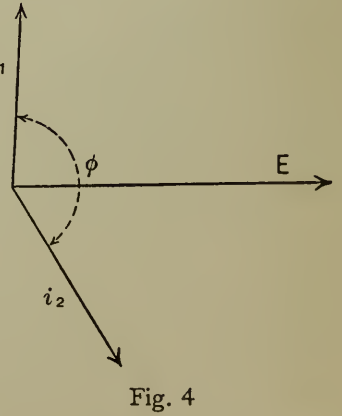

Fig. 4

The adjustment of the circuits is, in practice, best made as follows: Having adjusted the frequency roughly, a suitable inductance is found, such that, with the voltage used, when the circuit is

closed through the inductance, the fixed coils f_1 and the movable coils in series, a deflection of about 1 meter is produced. This current is next broken and a current passed through the fixed coils f_2, the movable coils, and a condenser variable in steps, all in series. The capacity of this latter circuit is adjusted until the deflection obtained has about the same magnitude as that previously obtained with the inductance in circuit. The fixed coils f_1 in series with the inductance coil are then shunted between the terminals of the circuit containing the condenser, care being taken that the two torques due to the action of the two currents are in opposition. It is now easy by continuous variation of the capacity or the inductance, or both, to bring the dynamometer deflection to zero. By increasing the impressed voltage, the sensibility may be increased within limits compatible with the safe current carrying capacity of the movable coils of the dynamometer. The correct frequency for resonance with the galvanometer having been reached and held constant by the use of a variable resistance in the field circuit of the alternator, a second observer balanced the dynamometer circuits. He was thereafter in a position to keep the frequency at the correct value (by manipulating the resistance in the motor field circuit), while the other observer made the settings on the alternating current bridge.

With this method of speed control, the resistance of the coil L does not have to be kept negligibly small, as was assumed for the sake of the demonstration, but the angle of lag of this circuit should be kept large enough to make the phase difference between i_1 and i_2 (Fig. 4) considerably greater than 90°, in order that the torque t may not be unduly decreased by the approach to a condition of quadrature of the two currents.

The actual value of the resonance frequency was, in the present instance, sufficiently well determined by measuring the speed of the alternator by means of a speed counter. Greater precision may, however, be attained, when desired, by the use of an auxiliary bridge containing resistances in three of its arms, and a known capacity in series with a calibrated variable inductance in the fourth arm. The frequency being held at the value which is to be measured, by means of the dynamometer arrangement, a

second observer balances the auxiliary bridge (which is supplied by the same source) by alternately varying the inductance and resistance of its fourth arm. For this measurement the same vibration galvanometer is used as in the main measurements, its frequency of vibration being of course the frequency which it is desired to determine. The frequency is calculated from the capacity and inductance necessary to balance the auxiliary bridge, using the familiar relation $p^2LC = 1$. This extremely sensitive method for determining the value of a frequency is evidently applicable to any case where it can be maintained constant by auxiliary apparatus. The most perfect arrangement is to use for detecting the balance of the auxiliary bridge a vibration galvanometer whose period of resonance is adjusted to the period of the current whose frequency is to be measured, but with a fairly pure wave form, a telephone could probably be used where a moderate sensibility is sufficient.

4. CONTROL OF THE TEMPERATURE

The condensers were kept in a galvanized iron tank 58 x 50 x 50 cm surrounded by a water jacket on all sides, except the top, which was insulated by a layer of hair felt 3 cm thick. The quantity of water in the jacket (about 100 liters) was so large that its temperature when within 5° of the temperature of the room, changed very little from day to day, requiring thus little or no regulation. The extreme temperatures, 10° and 35°, were obtained by filling the water jacket with water whose temperature was about 3° lower or higher, respectively, than the desired temperature, about 24 hours before it was desired to make the measurements. From previous experience this was believed to be long enough to insure the condensers taking up the temperature of the bath which after this interval was at closely the temperature desired. Since in every instance check measurements, made after an interval of several days, agreed well with those made 24 hours after adjusting the temperature of the bath, this assumption was undoubtedly sufficiently well justified. Owing, however, to the creeping of the temperature of the jacket at 10° and 35° during the four or five hours, required for a complete set of measurements, the temperature of the condensers was

uncertain by a few tenths of a degree, at these extreme temperatures, which may perhaps be a sufficient explanation for the fact that the curves do not repeat themselves perfectly at 35°, where uncertainties in the temperature are of the greatest effect. The results are, however, not seriously affected by this source of error. Measurements were made at two frequencies each evening, but owing to the number of condensers studied, two evenings were required to finish the list in the earlier work where the Series Inductance Method was employed. The use of the Series Resistance Method in the later series reduced materially the time required for the measurements. In the work at 33 and 100 cycles, the temperatures followed the order 19°, 25°, 30°, 34°, 20°, 15°, and 10°. The values observed at 20°, both with ascending and descending temperatures, agreed so well, that in the later series at the higher frequencies, measurements were made at 15°, 25°, and 35° only, without repetition. Settings were made at 100 cycles in every set, so as to give a check on the constancy of the condensers during the progress of the work, as well as to more accurately obtain the effect of frequency, unaffected by any correction for temperature. Most of the condensers showed a constancy over the interval of seven months between the first and second parts of the work, which was better than had been expected.

5. CONDENSERS STUDIED

The following table summarizes all of the data available on the details of construction of the condensers tested. In all except the condensers designated as $Be_{1,5}$ and $Be_{3,4}$ the dielectric is paraffined paper; in the two exceptions the paper had been impregnated with beeswax.

Of these condensers S_3, S_6, and S_{10}, t_3, t_{17}, and W_a each is completely sealed in the inclosing metal case, the terminals of the first group being brought out through insulating bushings. t_3, t_{17}, T_2, and W_a were especially designed for telephone work; the others for general laboratory use.

None of these condensers are clamped. In *Cp*, however, the space between the condenser and the cover of the box is occupied by strips of cardboard. The condenser *Mp* quite accurately fills its containing box, and in *Lp* two blocks of wood have been

introduced, which fit rather tightly between the condenser and the cover of the box. *SH* does not fit its case at all tightly. In view of these facts it may be remarked in passing that the majority of the condensers suffered permanent changes of capacity or phase difference, or both, during the observations. These changes were for the most part unimportant. Of the condensers, T_2 and W_a showed changes in the capacity only, W_a to about 0.1 per cent

TABLE I

Designation	Where made	Box	Construction	Paraffin		
				Color	Softened	Melted
S_3, S_6, S_{10}	U. S.	Metal	Flat sheets, 25 cm x 30 cm	White	(?)	(?)
Cp	France	Wooden	Flat sheets, 25 cm x 30 cm	Gray	36°	52°.5
Lp	U. S.	Wooden	Flat sheets, 15 cm x 20 cm	Yellow	30°(?)	51°
Mp	England	Wooden	Roll, 20 cm wide, four sections	Yellowish white	48°	57°
SH	Germany	Wooden	Sheets, 15 cm square	Very white	48°	58°
T_2	U. S.	Metal	Paper and tin foil wound together on wood	Dark gray	32°	48°
W_a	U. S.	Metal	Strip, 10 cm wide, rolled up			
$Be_{1,5}$, $Be_{3,4}$	U. S.	None	Sheets, 10 cm x 15 cm			
t_3, t_{17}	U. S.	Metal	Strip, 8 cm wide, rolled up			

and T_2 a permanent decrease of about 1 per cent. $Be_{1,5}$ exhibited a decrease in the phase difference of perhaps 10 per cent at the lower frequencies, but no noticeable change in capacity. Of the remaining condensers, the following permanent changes were noticed: *SH*, increase in capacity 6 in 10 000, increase in phase difference of about 5 per cent; *Cp*, increase in capacity 12 in 10 000, increase in phase difference about 10 per cent; S_6, a

decrease in phase difference of about 6 per cent, a decrease in capacity of 0.1 per cent, noticeable at 100 cycles only.

There are two cases which may occur, viz, a mechanical change in the condenser, such as a change in the configuration of its parts, due to a springing of its plates, while the dielectric is somewhat plastic at the higher temperatures, and second, some change in the nature of the dielectric. Of these two effects, the first should not involve the phase difference, but, on the other hand, any change in the phase difference tends to produce a change in the measured capacity, an increase in phase difference giving rise in general to an increase in the capacity. In a number of the above instances, these effects are qualitatively substantiated, although occasionally the effect of a change of phase difference seems to have produced no measureable influence on the capacity. The hypothesis of a change in capacity due to the relieving of stresses while the dielectric is somewhat plastic gains probability in the light of the comparatively low softening point observed for some of the samples of paraffin taken from these condensers (see Table I), although the observed changes do not follow very closely these observed temperatures of noticeable plasticity.

6. METHOD OF PRESENTING THE RESULTS

The following table of the results of a single day's observations at a single frequency, will illustrate the magnitudes of the observed quantities, and serve to indicate the concordance obtained with different values of the ratio arms.

In these observations the various sections of the mica condenser *B*, whose capacity and phase difference at various temperatures and frequencies had previously been very accurately determined, were made the basis of the work. The condensers 7763 and 7766 were also used in conjunction with the former, and since these last condensers had not been previously studied at the higher frequencies, their values were connected with those of condenser *B* by direct comparison on the day of observation (as in the above table).

In Table II the capacities in column 4 were calculated from the observed values of R_3 and the known value of the standard, which is placed last in each group of comparisons. From the observed values of r_1 in the sixth column the differences in phase

referred to the phase difference of the standard were calculated by formula (4) and are tabulated in the seventh column. The last column contains the absolute values of the phase differences of the condensers, computed from the known phase differences of the standards.

TABLE II

July 12, 1907. Frequency 314 cycles. Temperature 25°.8

Condensers		R_3	Capacity (observed) mf	r_1 ohms	$\phi_1'-\phi_1''$	Phase diff.
Auxil.	Test					
					° ′ ″	° ′ ″
L	S_{10}	598.50	3.03727	2.474	11 31	
+7764	S_{10}	1098.90	30	2.508	48	13 0
+7765						
L	S_6	575.06	3.14525	0.765	46 3	47 24
+7764	S_6	1061.16	29	0.806	6	
+7765	7766 +B+ 7763	600.33 1107.80	(3.01286)	3.049 3.094		(1 20)
7764	S_3	599.86	1.60952	1.933	12 55	
$+L_{06}$	S_3	1106.97	51	2.007	13 6	14 10
	7766 $+B_{0.6}$	600.49 1108.12	(1.60784)	3.118 3.208		(1 10)
7764	t_{17}	567.34	1.91462	2.356	3 36 9	
$+L_{0.7}$	t_{17}	1047.08	38	2.493	38 43	3 38 0
	T_2	610.40	1.77959	15.007	65 39	
	T_2	1126.46	48	15.066	51	66 53
	7766 $+B_{0.8}$	600.56 1108.25	(1.80871)	20.109 20.184		(1 8)
L	SH	591.70	1.02207	5.226	34 42	
L	SH	1091.91	06	5.362	48	35 45
L	Mp	615.46	0.98262	9.721	6 6	7 6
L	Mp	1135.78	58	9.771	53	7 53
L	Lp	628.08	0.96289	6.552	28 8	
L	Lp	1159.11	80	6.710	9	29 8
L	7766	602.41	(1.00391)	10.410		(1 0)
L	7766	1111.65		10.563		

TABLE II—Continued

July 12, 1907. Frequency 314 cyles. Temperature 25°.8—Continued

Condensers		R_3	Capacity (observed) mf	r_1 ohms	$\phi'_1-\phi_1''$	Phase diff.
Auxil.	Test					
					° ′ ″	° ′ ″
L	W_a	587.47	1.02940	6.221	27 11	
L	W_a	1084.14	31	6.361	7	28 26
L	7763	602.24	1.00411	10.302	0 24	
L	7763	1111.36	10	10.436	24	1 44
L	7766	602.36	1.00390	10.410	−0 19	
L	7766	1111.56	92	10.558	− 15	1 0
L	B	601.80	(1.00485)	10.354		(1 17)
L	B	1110.52		10.488		
$L_{0.6}$	t_3	542.35	0.66868	6.9	9 12 0	9 37 0
$L_{0.6}$	t_3	1005.35	569	10.4	28 0	
$L_{0.6}$	$B_{0.6}$	600.50	(0.60393)	150.763		
$L_{0.6}$	$B_{0.6}$	1108.16		150.994		(1 17)
$L_{0.5}$	$Be_{1,5}$	530.40	0.56893	3.237	58 48	
$L_{0.5}$	$Be_{1,5}$	1067.07	98	3.513	40	59 44
$L_{0.5}$	$Be_{3,4}$	566.91	0.53230	7.355	44 43	
$L_{0.5}$	$Be_{3,4}$	1046.13	31	7.623	45	45 44
$L_{0.5}$	Cp	623.28	0.48415	12.615	29 50	
$L_{0.5}$	Cp	1150.24	12	12.916	50	30 50
$L_{0.5}$	7766_r	601.32	(0.50184)	20.921		(1 0)
$L_{0.5}$	7766_r	1109.64		21.211		
$L_{0.5}$	7766_r	1209.83		21.194		

It has seemed most practicable and conducive to clearness not to tabulate the whole mass of the observed data, but rather to embody the results in curves, which shall give, along with the numerical values, a clear picture of the variations of the observed quantities with varying temperature and frequency. Accordingly, the sum of the results on each condenser is incorporated in two sets of curves, which present (a) the capacity and phase difference as functions of the temperature, with the frequency as variable parameter, and (b) the capacity and phase difference as functions of the frequency, with the temperature as variable parameter. To

distinguish between the curves of capacity and phase difference, the latter are drawn as dotted lines, while the curves of capacity are in full lines. These 26 sheets of curves have been numbered for ready reference.

Unfortunately the differences in the values of the constants of the various condensers are of such widely different magnitudes as to make it impracticable to draw all the curves to a uniform scale, without in certain instances obscuring points of interest. The scale of each curve is, however, clearly indicated, and to lend clearness and emphasize the relations between the different scales, the curves for the mica condenser *B* are drawn to scale on each sheet.

7. CURVES OF RESULTS

A casual inspection of the curves of figures 5 to 30 suffices to show how greatly the different condensers vary among themselves in their constants. The relations connecting the capacity and phase difference with the temperature and frequency are far from simple, and their detailed consideration in the light of the various theories of absorption will be taken up in the second part of the paper. The following points are, however, of importance from a practical standpoint.

(1) The **changes of capacity** with the frequency and temperature are much larger than are observed with mica condensers, and are very appreciable in every case. The amount of change is in general larger with those condensers which have a large phase difference. In the case of the poorer condensers studied (Figs. 21–30) the changes of capacity, referred to the value at the highest frequency and lowest temperature, where the effects of absorption are smallest, range from 5 to 60 per cent, for the temperatures and frequencies studied.

(2) The **phase differences** observed range from about 6′ in the case of *Mp* (Figs. 5 and 6) to the enormous value of about 22° with the condenser t_3 (Figs. 29 and 30). The phase difference of a good mica condenser may be as low as 15″ at the higher frequencies, and usually lies under 3′ even at the higher temperatures and lower frequencies.

(3) The **temperature coefficient** of the capacity is in some cases negative and nearly constant (Figs. 5, 7, and 9) and in others

positive and rapidly increasing with the temperature (Figs, 15, 17, 21, 25, 27, and 29). In still other condensers the temperature coefficient is negative at the lower temperatures and higher frequencies, and tends to become positive with increasing temperature, especially at the lower frequencies (Figs. 17, 19, and 25). The negative temperature coefficient occurs as a rule with those condensers having small phase differences, and the coefficient, although larger than that of a good mica condenser, does not exceed about 5 parts in 10 000. On the other hand, the positive temperature coefficients, which are usually found in condensers having large phase differences, are in many cases of the order of 1 per cent at the higher temperatures and lower frequencies.

(4) **Change of capacity with frequency.**—With increasing frequency the capacity falls off, at first rapidly and then more slowly, and tends to reach a limiting value, asymptotically, which represents the value of the capacity as unaffected by absorption. With a given condenser these changes with the frequency are larger at the higher temperatures, and in the case of condensers with large phase difference may become very large (Figs. 22, 24, 26, 28, and 30).

(5) **Change of phase difference with frequency.**—In the majority of cases the phase difference increases with rising temperature, but whereas the change is small and nearly linear in the case of mica condensers, paper condensers show as a rule a rapid rise of the curve at the higher temperatures and lower frequencies, and with some of the poorer condensers here studied these effects are enormous. The change of the phase difference with the frequency will be considered more in detail below.

The above considerations will make clear the fact that paper condensers are, at best, poorly adapted to serve as standards of capacity, and even in work of small precision should be excluded, unless a preliminary study has shown that the capacity is sufficiently constant over the range of temperatures and frequencies in which it is to be used.

From the paper of Curtis[9] it appears that the capacity of mica condensers when tested with direct current is far more dependent on the conditions of the experiment than with

[9] This Bulletin, **6**, p. 457; 1910.

alternating current. It would therefore appear from the foregoing curves with alternating current, that the use of paper condensers as standards in measurements with direct current, and in fact in all direct-current work where a knowledge of the capacity is desired, is to be condemned. This conclusion has been abundantly borne out in work with the ballistic galvanometer at the Bureau of Standards and elsewhere. It is no uncommon experience to find condensers whose apparent capacity with periods of charge and discharge of several seconds is double that observed with a quick charge and discharge, while the value under the same conditions of charge and discharge increased with every successive charge. These effects are of course smaller in any given case with short periods of charge and discharge, but the results of Prof. Zeleny (Phy. Rev., 1911) with charging and discharging periods of only a few thousandths of a second, show that even then the capacity and temperature coefficient are appreciably affected by the conditions of the experiment.

8. EFFECT OF RESISTANCE EXTERNAL TO THE CONDENSER

The curves connecting the values of the capacity at different temperatures and frequencies are so smooth as to indicate that the errors of observation are of small effect. The curves showing the change of phase difference with the frequency are, on the contrary, rather irregular; instead of falling off asymptotically with increase of frequency, as is the case with mica condensers, the phase difference in some cases actually increases with the freqeuncy over a moderate range, and begins to fall off again, if at all, only at the highest frequencies. Although in some of these cases, as we shall see later, there is evidence to show that the phase difference should be larger at frequencies of the order of 300 cycles than at 100 cycles, this form of the curve is probably mainly due in the majority of cases to the effect of the internal resistance of the condenser.

As already stated, the resistances of the leads from the condenser to the bridge were carefully measured, and a correction applied to the measured value of the phase difference. If, however, there be in addition an appreciable internal resistance in the condenser, the measured values of the phase difference will still be too large, by a value which is proportional to the frequency, to the value of

this resistance, and to the capacity of the condenser. This correction to the phase difference is given by the formula $\tan \theta = pC\rho$ as before. Trial has shown that in the case of most of the curves an approximate value of the resistance ρ may be found, which will give a corrected curve of the phase difference with the frequency, falling rapidly at the lower frequencies, but decreasing asymptotically at the higher frequencies. The following table shows the values thus found, together with the specification of the curves from which they were derived:

TABLE III

Condenser	Temperature	ρ ohms
SH	15° and 25°	0.15
Mp	15°, 25°, and 35°	0.07
t_{17}	15° and 25°	1.0
$Be_{3.4}$	15°, 25°	1.0
T_2	15°, 25°	0.25
Lp	15°	0.50
Lp	25°, 35°	0.20
Cp	15°, 25°	1.3
S_6	15°, 25°	0.25
W_a	15°, 25°, 35°	1.92
S_3	15°, 25°	0.21
S_{10}	35°	0.08
S_{10}	25°	0.20
t_3	15°, 25°	3.6
$Be_{1.5}$	15°, 25°	0.7

These values of ρ are in most cases quite definite. It seems nevertheless impossible to derive values of ρ for Mp, S_{10}, and S_3, Figs. 5–10, which shall hold for all the curves and for each curve over the whole range of frequency. It is just these condensers, however, for which there is theoretical evidence that the shape of the curves of phase difference φ with the frequency is more complicated than the simple form here assumed.

This extra resistance ρ has been calculated on the assumption that it is entirely due to causes external to the plates of the condenser, viz, lead resistances between the binding posts and the

plates, and contact resistances where the tin foil is welded together and joined to the leads. Since, with the most of these condensers, the plates are all in parallel, the plate resistance should be practically negligible. The three telephone condensers t_3, W_a, and t_{17}, Figs. 29, 30, 11, 12, 27, and 28 are made by rolling up together two or more sheets of tin foil with the dielectric between them. Since the number of sheets in parallel is small, and each sheet is very long and thin the plate resistance should form an appreciable part of the whole resistance, and this seems to be borne out by the large value of ρ found in these cases. This explanation is confirmed by some recent measurements of Curtis on telephone condensers, all from the same maker, in some of which connection was made at frequent intervals along the edge of the plates. The phase difference in this latter case decreased with increasing freqeuncy, while with those in which the leads were attached to the ends of the plates only, the phase difference was considerably larger at the higher frequencies than at the lower.

The importance of the internal resistance of a condenser in shifting the phase does not seem to have received general attention. In many cases the energy loss in this resistance becomes of greater importance than that in the dielectric. (In the case of the condenser W_a, Fig. 12, the I^2r loss in the plates and connections is, at a frequency of 900, about three times as great as the loss in the dielectric). Not only does this cause unnecessary heating of the condenser, with consequent change of its capacity, but the apparent phase difference increases rapidly with the frequency.

II. THEORIES OF ABSORPTION

The question of the explanation of the phenomenon of absorption has long occupied the attention of the physicists, and much has been published on the subject. In the small space here available it is, however, necessarily impossible to give a complete view of the theories which have been advanced, and the evidence which has been collected for or against these various hypotheses. The author has been obliged to confine his attention to what seem to be the most hopeful theories in the light of the present experiments, and would direct the reader who wishes to gain a more complete knowledge of the subject to the really admirable bibliography of the subject by von Schweidler (appended to his article on

Die Anomalien im Verhalten der Dielectrika, Ann. der Phys., **18**, p. 766; 1907).

1. MAXWELL'S THEORY OF A HETEROGENEOUS DIELECTRIC

Maxwell has shown [10] that a *homogeneous* dielectric, even if its specific resistance be comparatively small, should be free from absorption. If, however, the dielectric be of *heterogenous* composition, absorption is a necessary consequence. Maxwell treated the case of a dielectric made up of parallel strata of different dielectrics and proved that unless the product, Kr, of the dielectric constant K and specific resistance r is the same for all the strata, charges will collect at the planes of contact of the various strata, which are only dissipated gradually on successive discharges of the condenser. That is, the composite dielectric shows the ordinary phenomenon of absorption. Maxwell confined his investigation to the case of a constant applied emf. Rowland [11] extended Maxwell's investigation to the case of a sinusoidal emf, and showed (*a*) that the capacity should depend upon the frequency, and (*b*) the phase difference between the current and the applied emf should differ from the ideal 90° existing in a homogeneous dielectric by an amount which is different according to the frequency. Rowland merely stated his results without proof. His equation for the equivalent resistance of the condenser seems to be slightly in error. The author has indicated the derivation of this result below:

Let the dielectric be considered as composed of parallel plane strata 1, 2, 3, ... of thicknesses a_1, a_2, a_3, ... dielectric constants K_1, K_2, K_3 ..... specific resistances r_1, r_2, r_3, ... and let the electric field strengths, assumed constant inside each stratum, be X_1, X_2, X_3, ...

The emf on the condenser is then

$$e = a_1X_1 + a_2X_2 + a_3X_3 + \ldots\ldots$$

Maxwell's equations for the current density are

$$i = \frac{X_1}{r_1} + \frac{K_1}{4\pi}\frac{dX_1}{dt} = \frac{X_2}{r_2} + \frac{K_2}{4\pi}\frac{dX_2}{dt} = \ldots\ldots$$

[10] Elect. and Mag., I, § 328–330. [11] Scientific Papers, p. 297.

If we assume that the current obeys the sine law; that is, $i = I \sin pt$, where $p = 2\pi \times$ the frequency

$$I \sin pt = \frac{X_1}{r_1} + \frac{K_1}{4\pi}\frac{dX_1}{dt} = \frac{X_2}{r_2} + \frac{K_2}{4\pi}\frac{dX_2}{dt} = \ldots\ldots$$

The electric forces, therefore, satisfy the differential equation

$$\frac{dX}{dt} + \frac{4\pi}{Kr}X = \frac{4\pi}{K} I \sin pt.$$

The solution of this equation is

$$X = Ae^{-\frac{4\pi}{Kr}t} + \frac{\frac{(4\pi)^2}{K^2 r} I}{p^2 + \left(\frac{4\pi}{Kr}\right)^2} \sin pt - \frac{4\pi p I}{K\left[p^2 + \left(\frac{4\pi}{Kr}\right)^2\right]} \cos pt.$$

The emf on the condenser is therefore

$$e = a_1 A_1 e^{-\frac{4\pi}{K_1 r_1}t} + a_2 A_2 e^{-\frac{4\pi}{K_2 r_2}t} + \ldots\ldots$$

$$+ (4\pi)^2 I\left[\frac{a_1}{K_1^2 r_1\left\{p^2 + \left(\frac{4\pi}{K_1 r_1}\right)^2\right\}} + \frac{a_2}{K_2^2 r_2\left\{p^2 + \left(\frac{4\pi}{K_2 r_2}\right)^2\right\}} + \ldots\ldots\right] \sin pt$$

$$- 4\pi I p\left[\frac{a_1}{K_1\left\{p^2 + \left(\frac{4\pi}{K_1 r_1}\right)^2\right\}} + \frac{a_2}{K_2\left\{p^2 + \left(\frac{4\pi}{K_2 r_2}\right)^2\right\}} + \ldots\ldots\right] \cos pt.$$

The exponential terms become neglible in a very short time, so that we have in the steady régime

$$e = \left(\frac{4\pi}{p}\right)^2 I\left[\frac{a_1}{K_1^2 r_1\left\{1 + \left(\frac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \frac{a_2}{K_2^2 r_2\left\{1 + \left(\frac{4\pi}{K_2 r_2 p}\right)^2\right\}} + \ldots\ldots\right] \sin pt$$

$$- \frac{4\pi I}{p}\left[\frac{a_1}{K_1\left\{1 + \left(\frac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \frac{a_2}{K_2\left\{1 + \left(\frac{4\pi}{K_2 r_2 p}\right)^2\right\}} + \ldots\right] \cos pt.$$

We may therefore write $e = E \sin (pt - \psi)$ where $E^2 = M^2 + N^2$ and $\tan \psi = \frac{N}{M}$, and N and M are respectively the coefficients of $\sin pt$ and $\cos pt$.

What we have designated as the *phase difference* is $\phi = 90° - \psi$ and is given by

$$\tan\phi = pC\rho = \frac{M}{N}$$

$$= \frac{4\pi}{p} \frac{\dfrac{a_1}{K_1^2 r_1\left\{1+\left(\dfrac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \dfrac{a_2}{K_2^2 r_2\left\{1+\left(\dfrac{4\pi}{K_2 r_2 p}\right)^2\right\}} + \cdots}{\dfrac{a_1}{K_1\left\{1+\left(\dfrac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \dfrac{a_2}{K_2\left\{1+\left(\dfrac{4\pi}{K_2 r_2 p}\right)^2\right\}} + \cdots} \qquad (5)$$

where ρ is the equivalent resistance, in series with the capacity, which will produce this phase difference.

To find the capacity we remember that the component of the emf at 90° behind the current in phase is $\frac{I}{pC}$.

$$\therefore \quad \frac{1}{C} = 4\pi\left[\frac{a_1}{K_1\left\{1+\left(\dfrac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \frac{a_2}{K_2\left\{1+\left(\dfrac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \cdots\right]. \qquad (6)$$

For infinite frequency we obtain for the limiting value of the capacity C_o, unaffected by absorption

$$\frac{1}{C_o} = 4\pi\left[\frac{a_1}{K_1} + \frac{a_2}{K_2} + \cdots\right]. \qquad (7)$$

This has frequently been designated as the *instantaneous* capacity. Curtis [12] uses the term *geometric capacity*, a name which has much to recommend it.

The equivalent resistance of the condenser ρ may be derived directly from equations (5) and (6), or we may derive its value from the energy loss in the condenser. The latter is found by integrating the product of the instantaneous current and the component of the emf in phase with the current, over the cycle. Placing this equal to $I_e^2\rho$, where I_e is the effective current, we find the same value for ρ as by the first method, viz:

$$\rho = \left(\frac{4\pi}{p}\right)^2\left[\frac{a_1}{K_1^2 r_1\left\{1+\left(\dfrac{4\pi}{pK_1 r_1}\right)^2\right\}} + \frac{a_2}{K_2^2 r_2\left\{1+\left(\dfrac{4\pi}{pK_2 r_2}\right)^2\right\}} + \cdots\right]. \qquad (8)$$

[12] This Bulletin, 6, p. 471; 1910.

If r_1, r_2, . . are large, as would naturally be expected to be the case, we may expand the equations (5), (6), and (8) in powers of $\frac{1}{p}$;

$$\frac{1}{C}=4\pi\left[\left(\frac{a_1}{K_1}+\frac{a_2}{K_2}+\ldots.\right)-\left(\frac{4\pi}{p}\right)^2\left(\frac{a_1}{K_1^3r_1^2}+\frac{a_2}{K_2^3r_2^2}+\ldots.\right)\right.$$
$$\left.+\left(\frac{4\pi}{p}\right)^4\left(\frac{a_1}{K_1^5r_1^4}+\frac{a_2}{K_2^5r_2^4}+\ldots.\right)-\ldots.\right] \quad (9)$$

$$\rho=\left(\frac{4\pi}{p}\right)^2\left(\frac{a_1}{K_1^2r_1}+\frac{a_2}{K_2^2r_2}+\ldots.\right)-\left(\frac{4\pi}{p}\right)^4\left(\frac{a_1}{K_1^4r_1^3}+\frac{a_2}{K_2^4r_2^3}+\ldots\right)+\ldots. \quad (10)$$

Rowland gave the same equation for $\frac{1}{C}$; his equation for ρ is

$$\rho=\frac{1}{p^2}\left[\frac{a_1}{r_1K_1^2}+\frac{a_2}{r_2K_2^2}+\ldots\right]-\frac{(4\pi)^2}{p^4}\left[\frac{a_1}{r_1^3K_1^4}+\frac{a_2}{r_2^3K_2^4}+\ldots\right]+\ldots$$

Hess [13] treated the special case of a pure capacity in series with a second capacity joined in parallel with a resistance. That is, he considered that an imperfect dielectric could be regarded as equivalent in its effect to a stratified dielectric, in which one of the two strata possesses an infinite specific resistance. He derived the equations for this case supposing a steady emf to be applied, and made experiments which seemed to support this theory.

It does not seem possible, on account of the number of variables involved, and the uncertainty as to how many and what kind of strata are present, to submit this theory to a quantitative test. *Qualitatively* there is, with a few possible exceptions, good agreement between the observations and the theory, as will next be shown.

With decreasing frequency, the capacity and phase difference should increase. The observations show, however, that the rate of increase is, in general, slower than would be demanded by only two terms in (5) and (6), unless r_1, r_2, . . are much smaller than would naturally be expected. It should, however, be possible, by assuming the number of strata large enough, to represent the

[13] Lum. Elect., **46**, p. 401, 507; 1892.

observations. Curtis has shown in a recent article,[14] that assuming the dielectric of the mica condensers investigated by him to consist of a stratum of mica and a stratum of paraffin (used for sealing the condenser) and taking for the values of a_1, a_2, r_1, r_2 those found by direct measurement, the variation of the capacity and phase difference calculated from the above equations is much smaller than that actually found.

The curves connecting the capacity and phase difference with change of frequency are of the same general form for all of the condensers here investigated, the changes being more pronounced the greater the order of magnitude of the phase difference of the condenser. This can be reconciled with the assumption that r_1, r_2, .. are smaller in the condensers with the greater absorption. A consideration of the temperature curves is, however, more instructive.

The thicknesses of the strata a_1, a_2, . . . will in general increase with rising temperature. The coefficient of expansion of paraffin is known to increase rapidly as the temperature increases. The following values, taken from Landolt and Börnstein's tables, will illustrate this point:

Temperature	Average coefficient
0°–16°	0.00058
16°–38°	0.00070
38°–49°	0.00258
33°.5–37°.7	0.00260
37°.7–41.0°	0.00666

The capacity per unit area of the plates (to which the foregoing equations apply) should, therefore, as a result of the expansion of the paraffin, show a negative temperature coefficient, which should increase more and more rapidly as the temperature increases. On the other hand, the area of the sheets of the dielectric should increase with the temperature, thus tending to give a positive temperature coefficient, due to this cause. In

[14] This Bulletin, 6, p. 474; 1910.

most mica condensers which have been sealed with paraffin, and in the better paraffined paper condensers such as Mp, S_3, and S_{10}, the former effect predominates and the resultant temperature coefficient is negative, and becomes rapidly more negative with increasing temperature, in accordance with the change in thickness of the strata.

There seems to be a scarcity of data as to the effect of temperature changes on the dielectric constant. Mica has, according to Cassie [15] a small positive temperature coefficient. On this assumption, the effect of changes in the dielectric constant should be to give rise to a positive temperature coefficient of the capacity. It is possible that the very small positive temperature coefficients observed by Curtis [16] in mica condensers from which most of the paraffin had been squeezed out, and in silvered mica condensers generally, may be explained by a preponderance of this effect over that of changes of the thickness of the strata. This gains credibility from the fact that the capacity varies directly with the dielectric constant. Equation (5) shows that a positive temperature coefficient of the dielectric constant should give rise to a negative temperature coefficient of the phase difference. This assumes, however, that changes of r_1 and r_2 with the temperature are of less effect, which is not usually to be expected.

The magnitude of the specific resistances r_1, r_2, . . . is, however, according to this theory, perhaps the most important factor in the behavior of paraffined paper condensers. The temperature coefficient of insulation resistance is known to be negative, the value increasing rapidly with the temperature. Further, the temperature coefficient is, in general, greater for small values of r.

Bearing these points in mind, we are in a position to analyze the temperature curves of capacity and phase difference for the various condensers. Mp, S_3, and S_{10} may be considered as constituting one group. As already remarked, the temperature coefficients of their capacities are negative and nearly constant at the lower temperatures, but become rapidly more negative above

[15] Proc. Roy. Soc., **46**, p. 357; 1889. [16] This Bulletin, **6**, p. 448; 1910.

about 25°. The condenser *Mp* has the largest coefficient, S_{10} the smallest. The effect of changes in r_1, r_2, ... is to tend to make the temperature coefficient positive, especially at the higher temperatures and lower frequencies. This effect is noticeable in S_3 and *Mp*. The phase difference of the latter is unusually small for a paper condenser, indicating relatively large values of the specific resistances, and consequently only small changes of phase difference with the temperature. In both cases, the tendency is for the temperature coefficient of the phase difference to change from negative, at the lower temperatures, to positive, at the higher frequencies, the temperature of the transition lying at a lower value, the lower the frequency. This can be explained, qualitatively at least, by the assumption that at the lower temperatures the effects of the decrease of capacity and increase of dielectric constant tending to cause a negative temperature coefficient of phase difference are together larger than the effect of the change in the specific resistances. As the temperature increases, the coefficients of r_1, r_2, . . increase (more rapidly at the lower frequencies than at the higher), until the effect of the specific resistance preponderates. The phase difference is larger with S_3 than with *Mp*, indicating smaller values of the specific resistances in S_3 than in *Mp*. Consequently larger changes of the phase difference with the temperature would be expected in the former than in the latter, as was observed. The interpretation of the curves of S_{10} is difficult; it requires, perhaps, the assumption that in this condenser the temperature coefficient of the dielectric constant is more important than in the other two condensers. This would give rise to the relatively small temperature coefficient of the capacity, the effect being somewhat annulled at the lower frequencies, as already touched upon. This assumption finds support in the unusually large temperature coefficient of the phase difference, even at the lower frequencies, although the magnitude of the phase difference would lead one to anticipate as great effect of the specific resistance as with S_3.

The condensers *Cp*, *Lp*, $Be_{1,5}$, $Be_{3,4}$, t_3, t_{17} group themselves naturally in another class from those just considered. These are distinguished by positive temperature coefficients of capacity, the

value of this coefficient increasing rapidly with the temperature, and with decreasing frequency. The phase difference curves are of the same character. The changes of both capacity and phase difference are on a much larger scale than those of the condensers already considered, and the actual magnitudes of the phase differences are in some cases excessive (compare those of t_3, t_{17}, $Be_{1,5}$, etc.).

According to the theory of a heterogeneous dielectric, the effect of the specific resistance, although in some of these cases nearly balanced at the lower temperatures and higher frequencies, must be the preponderating factor.

The drop in the phase difference curves of *Cp* at 30°–35°, and at the frequencies 33 and 100 is due to an actual permanent change taking place in the dielectric, which manifested itself by a permanent shift of the capacity curve at lower temperatures. In the curves of t_3, and, to a less extent, in those of t_{17}, the effect of the second term in the equation for ρ, which works in opposition to the main term, may perhaps be detected.

The remaining condensers partake, more or less, of both of the classes already discussed. *SH* shows very well the gradual change with the frequency of the factor which is predominant. The specific resistances of the strata are the principal factors in determining the temperature coefficient of the capacity. At 100 cycles the effect of the specific resistance is nearly balanced against that of the change in the thickness of the strata, the latter effect being the larger at the lower temperatures and the former at the higher. Finally, at 900 cycles the effect of the specific resistance has become so small that the curve has the distinctive shape of that produced by changes in the thicknesses of the strata alone. In the case of the phase differences the curves are what we would expect, the effect of the specific resistance being smallest at the highest frequency. The curves of S_6 are of the same nature as those of the preceding.

The condenser W_a is peculiar. It belongs to a lot of telephone condensers having the smallest phase difference (when corrected for internal resistance) of any telephone condensers yet tested by the author. The internal resistance of the plates is, however, large. The capacity temperature coefficient is positive and small

(about 3 in 10 000). The bending of the curves in the region above 30° is to be attributed to a tendency of the capacity to become permanently lower at the higher temperatures. The phase difference changes but little with the temperature, but the observed curves taken in conjunction with the capacity curves, and the smallness of the value of the phase difference, perhaps indicates an unusually large influence of the dielectric constant.

Finally, the condenser T_2, which was markedly unstable at higher temperatures, resembles otherwise *SH* and S_6, although the effect of the specific resistance was larger with the latter condenser than with the former.

We thus see that *qualitatively* most of the peculiarities of all the observed curves may be explained by equations (5) and (6) in the light of assumptions which must, in the main, be qualitatively correct, although in some cases open to doubt. Undoubtedly the existence of heterogeneity in the dielectric gives rise to absorption, but whether this is even the *main* cause for the absorption is at least questionable. Several distinctive features of the curves of S_3, S_{10}, and *Mp* seem to point to the presence of other factors. In any case, the theory, in the light of our present knowledge, does not seem capable of quantitative proof.

2. HOULLEVIGUE'S THEORY OF FRICTIONAL RESISTANCE

In two articles entitled "Sur l'échauffement électrique des condensateurs" [17] and "Sur les theories du residu électrique" Houllevigue has elaborated Hopkinson's theory [18] of residual elasticity in terms of Maxwell's theory.

He assumes the displacement in an imperfect dielectric to consist of two parts—(*a*) an instantaneous displacement of the ether until the elastic forces between the ether and the molecules are in equilibrium and (*b*) an additional gradual yielding of the molecules under this tension, so that to the instantaneous displacement of the ether is added the displacement of the mean position of the molecules.

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[17] Jour. de Phys. (3), **6**, pp. 113 and 120; 1897.

[18] Phil. Trans., **167**, p. 599; 1877.

In viscous media the forces which determine the displacement depend not only on the magnitude of the displacement, but on the velocity with which it takes place; that is, there is a frictional reaction proportional to the velocity of the displacement which manifests itself in the heating observed in imperfect condensers. Houllevigue has formulated this theory both for the case of a steady and for a sinusoidal applied emf. His equations for the latter case, somewhat amplified and with several misprints corrected, are reproduced below.

(a) 1ST CASE. DIELECTRIC WITHOUT ABSORPTION

Let Δ = the displacement at any moment of the ether
Φ = the field strength
ρ = the absolute density of the ether
e = the instantaneous difference of potential between the plates
d = the distance between the plates
S = the area of the plates
F = the elastic reaction of the ether, taken proportional to the displacement.

We have, then,

$$\Phi = \frac{e}{d}, \qquad F = \frac{4\pi}{K}\Delta.$$

The displacement must satisfy, therefore, the differential equation

$$\rho \frac{d^2\Delta}{dt^2} = \frac{e}{d} - \frac{4\pi}{K}\Delta.$$

If the impressed emf is E_0 and the resistance and inductance of the external circuit are respectively R and L, then

$$E_0 = e + Ri + L\frac{di}{dt} = e + RS\frac{d\Delta}{dt} + LS\frac{d^2\Delta}{dt^2}$$

where i, the current $= S\frac{d\Delta}{dt}$,

We have, therefore,

$$\left(L+\frac{\rho d}{S}\right)\frac{d^2\Delta}{dt^2}+R\frac{d\Delta}{dt}+\frac{4\pi d}{KS}\Delta=\frac{E_0}{S}.$$

$\frac{\rho d}{S}$ will usually be neglible in comparison with L, and, remembering that $Q=S\Delta$, $\frac{4\pi d}{KS}=\frac{1}{C}$, this is seen to be Thomson's classic equation. Putting $E_0=E \sin pt$, where $p=2\pi$ times the frequency and $L+\frac{\rho d}{S}=L'$, we have

$$L'\frac{d^2\Delta}{dt^2}+R\frac{d\Delta}{dt}+\frac{4\pi d}{KS}\Delta=\frac{E}{S}\sin pt,$$

which gives

$$\Delta=\frac{L'S(A^2+B^2)}{E}(A\sin pt-B\cos pt)+c_1e^{\alpha_1 t}+c_2e^{\alpha_2 t} \qquad (12)$$

where α_1 and α_2 are the roots of the equation $\alpha^2+\frac{R}{L'}\alpha+\frac{1}{L'C}=0$, and

$$A=\frac{1}{L'C}-p^2 \qquad B=p\frac{R}{L'} \qquad (13)$$

$C=\frac{KS}{4\pi d}$, the capacity of the condenser, and c_1, c_2 are constants.

Equation (12) gives for the steady régime

$$\Delta=\frac{E}{pS\sqrt{R^2+\left(L'p-\frac{1}{pC}\right)^2}}\sin(pt+\theta)$$
$$\tan\theta=\frac{R}{pL'-\frac{1}{pC}} \qquad (14)$$

and

$$i=\frac{dQ}{dt}=S\frac{d\Delta}{dt}=\frac{E}{\sqrt{R^2+\left(L'p-\frac{1}{pC}\right)^2}}\sin(pt+\varphi) \qquad (15)$$
$$\tan\varphi=\frac{\frac{1}{pC}-pL'}{R}$$

the usual equation for the current in a circuit containing resistance R, inductance L', and capacity C.

(b) 2ND. CASE, DIELECTRIC WITH ABSORPTION

In addition to the displacement Δ of the ether, we have here to assume a displacement Δ' of the molecules of the dielectric themselves. The forces acting on unit volume of the dielectric, are (*a*) the elastic force $\frac{4\pi\Delta}{K}$, due to the displacement Δ of the ether, (*b*) the elastic reaction $a\Delta'$ of the molecules, opposing the displacement of the molecules and proportional to it, (*c*) a frictional reaction $2b\frac{d\Delta'}{dt}$ proportional to the velocity of the molecules. Consequently, if P is the density of the dielectric,

$$P\frac{d^2\Delta'}{dt^2}=\frac{4\pi}{K}\Delta-a\Delta'-2b\frac{d\Delta'}{dt}. \tag{16}$$

To simplify the integration, and since we need not consider the external circuit, we put $R=0$, $L=0$, and insert for the displacement Δ its value

$$\Delta=\frac{E}{pS\left(p\frac{\rho d}{S}-\frac{1}{pC}\right)}\sin pt=\frac{E}{p^2\rho d-\frac{S}{C}}\sin pt$$
$$=A\sin pt.$$

The differential equation for Δ' becomes therefore,

$$\frac{d^2\Delta'}{dt^2}+\frac{2b}{P}\frac{d\Delta'}{dt}+\frac{a}{P}\Delta'=\frac{4\pi A}{KP}\sin pt$$

whose solution is

$$\Delta'=C_1e^{\beta_1 t}+C_2e^{\beta_2 t}+\frac{4\pi}{KP}A\,[M\sin pt+N\cos pt] \tag{17}$$

where β_1 and β_2 are the roots of the equation

$$\beta^2+\frac{2b}{P}\beta+\frac{a}{P}=0$$

$$M=\frac{\frac{a}{P}-p^2}{\left(\frac{a}{P}-p^2\right)^2+\frac{4b^2}{P^2}p^2} \tag{18}$$

$$N=-\frac{\frac{2b}{P}p}{\left(\frac{a}{P}-p^2\right)^2+\frac{4b^2}{P^2}p^2}$$

or in the steady state

$$\Delta'=A'\sin(pt+\theta)$$

where

$$A'=\frac{4\pi A}{KP}\frac{1}{\sqrt{\left(\frac{a}{P}-p^2\right)^2+\frac{4b^2}{P^2}p^2}}=Ax \tag{19}$$

$$\tan\theta=\frac{-\frac{2b}{P}p}{\left(\frac{a}{P}-p^2\right)}$$

The current is

$$\begin{aligned} i&=S\frac{d}{dt}(\Delta+\Delta')\\ &=SAp\,[\cos pt+x\cos(pt+\theta)]\\ &=SAp\,[(1+x\cos\theta)\cos pt-x\sin\theta\sin pt]\\ &=I\cos(pt-\varphi) \end{aligned}$$

where

$$I=SAp\sqrt{1+2x\cos\theta+x^2}$$

$$\tan\varphi=\frac{-x\sin\theta}{1+x\cos\theta}=\frac{\frac{4\pi}{KP}\cdot\frac{2bp}{P}}{\left(\frac{a}{P}-p^2\right)^2+\frac{4\pi}{KP}\left(\frac{a}{P}-p^2\right)+\frac{4b^2}{P^2}p^2} \tag{20}$$

The current in the case of the dielectric without absorption is $SAp\cos pt$. The increase in capacity ΔC due to absorption is

therefore given by

$$\frac{\Delta C}{C} = x \cos\theta = \frac{4\pi}{KP} \frac{\left(\frac{a}{P} - p^2\right)}{\left(\frac{a}{P} - p^2\right)^2 + \frac{4b^2}{P^2}p^2} \tag{21}$$

and the phase difference is equal to the angle φ just derived. From the equation

$$\beta^2 + \frac{2b}{P}\beta + \frac{a}{P} = 0,$$

we have

$$\beta = -\frac{b}{P} \pm \sqrt{\frac{b^2}{P^2} - \frac{a}{P}}$$

Excluding therefore the case of an oscillatory charge, β will be real and negative and $\frac{b^2}{P^2} > \frac{a}{P}$. The term $\frac{4\pi}{KP}$ will vary for different dielectrics.

An interesting consequence of the equation (21) for $\frac{\Delta C}{C}$ is that with indefinitely increasing frequency, ΔC must at length change sign and the capacity become less than the value of the capacity as unaffected by absorption, i. e., the geometrical capacity, and with still greater frequency, approach asymptotically the geometrical capacity.

The physical significance of the equations (20) and (21) may be seen from the following considerations. The quantity $\frac{a}{P}$ is equal to $4\pi^2/T^2$, where T is the natural period of vibration of the molecules, when acted on by the elastic reaction alone, friction being supposed negligible. Equations (19) show that the phase of the viscous displacement lags behind that of the applied electric field by an angle which increases with increasing frequency, and which is greater the greater the frictional resistance to the displacement of the molecules. When the frequency reaches a value equal to that of the natural frequency of vibration of the molecules under the elastic restoring forces $\left(\frac{a}{P} = p^2\right)$ equation (19)

shows that the viscous displacement Δ' lags behind the field by exactly 90°; the energy is all expended in supplying the frictional losses, and the amplitude of vibration is the same as would be the case in a perfect dielectric, since there is no component of the viscous displacement in phase with the field.

When $\left(\frac{a}{P} \gtrless p^2\right)$ the viscous displacement may be decomposed into two components, one in phase with the field, and the other at right angles with it. The latter is the component which overcomes the friction, while the former increases the ideal displacement or is subtracted from it according as $\frac{a}{P}$ is less than or greater than p^2.

There is no experimental evidence in support of such a law of change of capacity with the frequency.

From the nature of the observed curves of capacity, and from the fact that Curtis found good agreement between the limiting value of the capacity, with increasing frequency, as extrapolated both from direct current and alternating current measurements, it does not seem reasonable to suppose that (21) actually represents the facts. If, however, it is ever found that with increasing frequency the capacity reaches a certain minimum value, and then rises slowly to a limiting value, this very simple theory will receive a valuable support.

Beyond the condition $\frac{b^2}{P^2} > \frac{a}{P}$, the values of the constants in the formula (21) are unrestricted, but notwithstanding this freedom of choice of their values the observations do not appear to follow the law in question.

3. PELLAT'S THEORY

Pellat assumed [19] that, when a field of strength E_0 is impressed on a dielectric, the displacement instantaneously takes up the value $D = \frac{K}{4\pi} E_0$, where K is the dielectric constant, and then increases logarithmically toward a final value $(1 + \epsilon)$ times as great,

[19] Ann. Chim. et Phys., 18, p. 150; 1899.

ϵ being small compared with unity. That is

$$D_t = \frac{K}{4\pi} E_0 \left[1 + (1 - e^{-at})\epsilon\right] \tag{22}$$

and therefore

$$D_\infty = \frac{K}{4\pi} E_0 (1 + \epsilon).$$

Accordingly

$$\frac{dD_t}{dt} = \alpha e^{-at} \frac{\epsilon K E_0}{4\pi} = \alpha (D_\infty - D_t). \tag{23}$$

That is, the rate of increase of the displacement at any moment is proportional to the difference between the final value and that existing at the time in question. Such a law has been advocated by several different observers from work with direct currents. The following extension of this theory to the case of a sinusoidal emf is adapted from von Schweidler's valuable and comprehensive article on dielectric theory.[20]

For any law of variation of the electric force with the frequency $E_t = f(t)$,

$$D_t = \frac{KE_t}{4\pi} + D_t' \tag{24}$$

where the instantaneous displacement D'_t is subject to the relation

$$\frac{dD_t'}{dt} = \alpha (D_\infty - D_t) \tag{25}$$

D_∞ being the final value of the displacement which would be reached if the electric field were held constant at the instantaneous value E_t.

But from (22)

$$D_\infty = \frac{K}{4\pi} E_t \, (1 + \epsilon)$$

and hence and from (24) and (25)

[20] Ann. der Phys., **24**, p. 711; 1907.

$$\frac{dD_t'}{dt} = \alpha\left[\frac{K\epsilon}{4\pi}E_t - D_t'\right]$$

or

$$\frac{dD_t'}{dt} + \alpha D_t' = \frac{\alpha K\epsilon}{4\pi}E_t. \tag{26}$$

Assuming now $E_t = E_0 \sin pt$

$$\frac{dDt'}{dt} + \alpha D_t' = \frac{K\alpha\epsilon}{4\pi}E_0 \sin pt \tag{27}$$

whose solution is

$$D_t' = Ae^{-\alpha t} + M \sin pt + N \cos pt$$

where

$$M = \frac{\alpha^2\epsilon KE_0}{4\pi(p^2+\alpha^2)}$$

$$N = -\frac{\alpha p\epsilon KE_0}{4\pi(p^2+\alpha^2).} \tag{28}$$

The current is

$$i = \frac{dD_t}{dt} = \frac{K}{4\pi}\frac{dE_t}{dt} + \frac{dD_t'}{dt}$$

$$= \frac{pKE_0}{4\pi}\cos pt + \frac{dD_t'}{dt}$$

or in the steady régime

$$i = \frac{KE_0p}{4\pi}\left(1 + \frac{\alpha^2\epsilon}{p^2+\alpha^2}\right)\cos pt + \frac{KE_0}{4\pi}\left(\frac{p^2\alpha\epsilon}{p^2+\alpha^2}\right)\sin pt \tag{29}$$

or

$$i = I\cos(pt - \varphi)$$

where

$$\tan\varphi = \frac{\frac{p^2\alpha\epsilon}{p^2+\alpha^2}}{p\left(1 + \frac{\alpha^2\epsilon}{p^2+\alpha^2}\right)} = \frac{\alpha\epsilon p}{p^2+\alpha^2(1+\epsilon)} \tag{30}$$

$$I = \frac{KE_0}{4\pi}\frac{p^2}{p^2+\alpha^2}\sqrt{\left\{1 + \frac{\alpha^2}{p^2}(1+\epsilon)\right\}^2 + \alpha^2\epsilon^2}$$

Since the emf is in phase with the electric field, φ is the *phase difference.*

The current in a condenser without absorption, acted upon by a sinusoidal emf is, as we have seen, $\frac{dD_t}{dt}=\frac{KE_0p}{4\pi}\cos pt$. We find accordingly that

$$\frac{\Delta C}{C}=\frac{\alpha^2\epsilon}{p^2+\alpha^2} \tag{31}$$

These simple equations are not satisfied by the observations of this paper, and were also found insufficient by von Schweidler.

4. MODIFIED PELLAT THEORY OF VON SCHWEIDLER

Having found that the Pellat theory, although in qualitative agreement with the observed phenomena in dielectrics, did not represent his measurements quantitatively, von Schweidler went a step further, and made the assumption that the law of change of the absorbed charge, instead of being expressed by a single exponential term as in Pellat's theory, may be represented by a series of such terms, viz:

$$D_t=\frac{KE_0}{4\pi}\left[1+\Sigma\epsilon(1-e^{-\alpha t})\right]$$

and that as a consequence

$$\frac{\Delta C}{C}=\frac{\alpha_1^2\epsilon_1}{p^2+\alpha_1^2}+\frac{\alpha_2^2\epsilon_1}{p^2+\alpha_2^2}+\frac{\alpha_3^2\epsilon_3}{p^2+\alpha_3^2}+\ldots. \tag{32}$$

$$\tan\varphi=\frac{\alpha_1\epsilon_1p}{p^2+\alpha_1^2(1+\epsilon_1)}+\frac{\alpha_2\epsilon_2p}{p^2+\alpha_2^2(1+\epsilon_2)}+\ldots. \tag{33}$$

In its greatest generality the number of these terms may be supposed infinite, the constants α_i having values varying continuously between 0 and ∞. The factors ϵ_i corresponding will be functions of α, $f(\alpha)$, so that we may write

$$D_t=\frac{KE_0}{4\pi}\left[1+\int_0^\infty f(\alpha)\{1-e^{-\alpha t}\}d\alpha\right]$$

and consequently

$$\frac{\Delta C}{C} = \int_0^\infty \frac{\alpha^2 f(\alpha)}{p^2 + \alpha^2} d\alpha \tag{34}$$

$$\tan \varphi = \int_0^\infty \frac{\alpha p f(\alpha)}{p^2 + \alpha^2 \{1 + f(\alpha)\}} d\alpha \tag{35}$$

The function $f(\alpha)$ being determined from the observations. Viewed mathematically von Schweidler has thus introduced into Pellat's equation for the change of displacement with the time, a sufficient number of constants to represent the complicated empirical function derived from the observations. To meet the objection that this may appear as nothing more than "an artificial, purely mathematical fiction," von Schweidler goes on to show that his assumption is capable of a simple physical interpretation. This exposition of his theory is given so clearly and concisely by von Schweidler[21] that it has seemed well to reproduce a translation of the same at length below. He says.

The molecules, which according to the electron theory consist of complexes of ions, may be regarded as resonators, in that the ions when displaced from their equilibrium positions possess definite natural periods of vibration. The phenomena of normal and abnormal dispersion of electric waves, together with the accompanying phenomena of absorption, are treated from this standpoint in the modern theory.

In addition, now, to molecules whose ions possess a definite period of vibration and a definite damping, we may suppose others to be present whose damping is so great that, instead of oscillating, their motion is *aperiodically damped.* Under the influence of a suddenly applied constant electric field these approach a new equilibrium position in such a way that their deviation from the latter diminishes as the exponential function $e^{-\alpha t}$.

The molecular-physical signification of the Pellat theory is, according to this point of view, as follows: In addition to those molecules which as resonators of definite (very small) period of oscillation follow the relatively slowly changing electric fields without an appreciable difference of phase, there are also present in an anomalous dielectric still other molecules in which the displacement of the ions takes place aperiodically damped, the constant α of the above formula having the same value for all the molecules. The quantity ε shows in what ratio the flux of the displacement of these aperiodically damped molecules stands to that of the molecules whose motion is oscillatory.

In the modified theory the assumption is made that not only *one* kind of such aperiodically damped complexes of ions with a definite time constant α is present, but on the contrary that a large number of different kinds having different time con-

[21] Ann. der Phys., **24**, pp. 747–748; 1907.

stants α_i are concerned, the number per unit volume of each kind being proportional to the quantity ε_i.

Going a step further, the possible values of the damping may be regarded as varying continuously, and the function $\varepsilon = f(\alpha)$ shows according to what law these molecules are distributed as α varies continuously between o and ∞.

The decomposition of an empirically given function into a sum of simple exponential functions corresponds then physically to a separation of the effects which are produced by different groups of molecules of the same kind.

To completely determine the distribution ε of the aperiodically damped molecules as a function of the time constant α—that is, to determine the form of $f(\alpha)$ in the equations (34) and (35) from the observed curves of $\frac{\Delta C}{C}$ and tan φ is probably impossible, and von Schweidler seems to have made no comparison of his results (found with steady currents) with the theory just exposed. It appeared worth while, however, to the author of the present paper to attempt to determine a finite number of constants in the formulas (34) and (35), and the results obtained show the adequacy of a few terms of the modified Pellat theory to represent empirically the observations of this paper. Before giving the constants thus found for some of the condensers here studied we will consider more closely the method used in calculating the α's and ϵ's.

(a) METHOD OF CALCULATING THE CONSTANTS

We are met at the start with the difficulty that the capacity curves do not give us $\frac{\Delta C}{C}$ directly, since the limiting or geometrical capacity is not known. The first method which suggests itself is to calculate $\alpha_1, \alpha_2, \alpha_3, \ldots$ and $\epsilon_1, \epsilon_2, \epsilon_3, \ldots$ from the curve of the change of phase difference with the frequency, the $2n$ unknowns being determined from the $2n$ simultaneous equations formed from the $2n$ measured ordinates of the curve. The form of these equations is, however, too complicated to make this practically feasible even if these curves were free from the effect of internal resistance. The capacity-frequency curves give us the *differences* in the quantity $\frac{\Delta C}{C}$ with different frequencies, but the equations connecting these observed values with the α's and ϵ's are not simple enough to render hopeful the solution of the simultaneous equations formed from these observations.

The problem appears to be incapable of an exact solution without at least prohibitive labor. A number of methods for obtaining an approximate solution were tried, with the result that it seems best to assume, first, probable values of the α's and then to solve for the ϵ's from the capacity curves. The values found are then used to calculate the phase differences at varying frequencies, and the curve thus derived is compared with the observed curve. If necessary, the α's are corrected by trial and the solution repeated until the calculated values of the phase difference are in sufficiently good agreement with those observed.

In the scheme finally adopted the ordinates of the capacity curve were measured at the frequencies $33\frac{1}{3}$, 100, 300, and 900 cycles. From these the *differences* in the quantity $\frac{\Delta C}{C}$ is obtained for the frequency intervals $33\frac{1}{3}$–100, 100–300, and 300–900, thus giving three values δ_1, δ_2, δ_3. Now, supposing for the moment that we have only one term in the equation (32) for $\frac{\Delta C}{C}$ we find for frequencies corresponding to values p, ap, a^2p, and a^3p

$$\frac{\Delta_1 C}{C} = \frac{\alpha^2\epsilon}{p^2+\alpha^2} = \frac{\epsilon}{1+\frac{1}{x^2}}, \quad \frac{\Delta_3 C}{C} = \frac{\alpha^2\epsilon}{a^4p^2+\alpha^2} = \frac{\epsilon}{1+\frac{a^4}{x^2}}$$

$$\frac{\Delta_2 C}{C} = \frac{\alpha^2\varepsilon}{a^2p^2+\alpha^2} = \frac{\varepsilon}{1+\frac{a^2}{x^2}} =, \quad \frac{\Delta_4 C}{C} = \frac{\alpha^2\epsilon}{a^6p^2+\alpha^2} = \frac{\varepsilon}{1+\frac{a^6}{x^2}}$$

where $x = \frac{\alpha}{p}$. Therefore using only one term

$$\begin{aligned}
\delta_1 &= \epsilon\left(\frac{1}{1+\frac{1}{x^2}} - \frac{1}{1+\frac{a^2}{x^2}}\right) = D\epsilon \\
\delta_2 &= \epsilon\left(\frac{1}{1+\frac{a^2}{x^2}} - \frac{1}{1+\frac{a^4}{x^2}}\right) = D'\epsilon \qquad (37) \\
\delta_3 &= \epsilon\left(\frac{1}{1+\frac{a^4}{x^2}} - \frac{1}{1+\frac{a^6}{x^2}}\right) = D''\epsilon.
\end{aligned}$$

In the actual case chosen we may consequently write:

$$\begin{aligned}\delta_1 &= D_1\epsilon_1 + D_2\epsilon_2 + D_3\epsilon_3 \\ \delta_2 &= D_1'\epsilon_1 + D_2'\epsilon_2 + D_3'\epsilon_3 \\ \delta_3 &= D_1''\epsilon_1 + D_2''\epsilon_2 + D_3''\epsilon_3\end{aligned} \tag{38}$$

where the quantities D_1, D_2, D_3 are found from D by giving to x the successive values $x_1 = \frac{\alpha_1}{p}$, $x_2 = \frac{\alpha_2}{p}$, etc., and similarly the coefficients D_1', D_2', D_3'' and D_1'', D_2'', D_3'' are derived from D' and D''. These coefficients may be calculated and tabulated for various values of the x's, and such a table has been found to expedite the work greatly.

Having now assumed certain values of the α's, and therefore for the x's, we write out the equations (38), taking the coefficients D from the table, and solve for the values of ϵ_1, ϵ_2, and ϵ_3.

The phase differences φ_1, φ_2, φ_3, and φ_4 corresponding to the frequencies $33\frac{1}{3}$, 100, 300, and 900 cycles may then be calculated from the equations

$$\begin{aligned}\tan\varphi_1 &= \frac{x_1\epsilon_1}{1+x_1^2(1+\epsilon_1)} + \frac{3x_2\epsilon_2}{1+9x_2^2(1+\epsilon_2)} + \frac{9x_3\epsilon_3}{1+81x_3^2(1+\epsilon_3)} \\ \tan\varphi_2 &= \frac{3x_1\epsilon_1}{9+x_1^2(1+\epsilon_1)} + \frac{x_2\epsilon_2}{1+x_2^2(1+\epsilon_2)} + \frac{3x_3\epsilon_3}{1+9x_3^2(1+\epsilon_3)} \\ \tan\varphi_3 &= \frac{9x_1\epsilon_1}{81+x_1^2(1+\epsilon_1)} + \frac{3x_2\epsilon_2}{9+x_2^2(1+\epsilon_2)} + \frac{x_3\epsilon_3}{1+x_3^2(1+\epsilon_3)} \\ \tan\varphi_4 &= \frac{27x_1\epsilon_1}{729+x_1^2(1+\epsilon_1)} + \frac{9x_2\epsilon_2}{81+x_2^2(1+\epsilon_2)} + \frac{3x_3\epsilon_3}{9+x_3^2(1+\epsilon_3)}.\end{aligned} \tag{39}$$

In all cases except for condensers of large absorption it will be found that the ϵ's are small with respect to unity, so that tables may be formed of the coefficients $\frac{x_1}{1+x_1^2}$, $\frac{3x_2}{1+9x_2^2}$, etc., corresponding to different values of α_1, α_2, and α_3 with a consequent saving of time.

The whole difficulty lies in the determination of the α's. A rough approximation to their values may be obtained from the capacity-frequency curves, on the assumption that the change of

capacity at each point is represented principally by the changes of a single term in the expression (32) for $\frac{\Delta C}{C}$. For example, we may assume that the shape of the curve near $33\frac{1}{3}$ cycles is mainly dependent on the value of α_1 alone, the shape near 100 cycles on α_2 alone, etc. Measuring ordinates at frequencies corresponding to p, ap, and bp, we find from the curve their differences.

$$\delta_0 = \frac{x^2(a^2-1)}{(1+x^2)(a^2+x^2)}\epsilon \ , \ \delta_0' = \frac{x^2(b^2-a^2)}{(a^2+x^2)(b^2+x^2)}\epsilon$$

and

$$z = \frac{\delta_0}{\delta_0'} = \frac{a^2-1}{b^2-a^2}\ \frac{b^2+x^2}{1+x^2}.$$

Therefore

$$x^2 = \frac{\alpha^2}{p^2} = \frac{Mb^2-z}{\zeta-M}$$

where

$$M = \frac{a^2-1}{b^2-a^2}$$

On applying this process to the curves of this paper, it was found that the underlying assumption is only in a general way true. Fortunately, excepting for the value of the smallest time constants α_1, quite extensive changes in the α's produce relatively small changes in the calculated values of the phase differences. In the single instance of α_1, the rough values found by the above method from the capacity-frequency curve are invariably too large to give the observed value of the phase difference at the smallest frequency, 33 cycles. This is explained by the fact that the character of the capacity curve is changing so rapidly in this region of frequencies that if intervals of frequency be selected large enough to carry through the method already described, the value found corresponds to an actual frequency so much larger than that for which the value of α_1 is required that the calculated value is not applicable. It also appears that the curve of phase difference at the lower frequencies is more sensitive to changes in the value selected for α_1 than is the capacity curve. The natural

suggestion that the α's should be calculated from the shape of the phase difference curve did not prove to be of advantage, since the phase difference curve, in addition to the errors of its determination, is subject to an uncertain error due to the internal resistance of the condenser, as already pointed out (p. 517).

The determination of the constants reduces, therefore, to a process of cut and try. The α's were obtained roughly, as already described, from the capacity curve, the simultaneous equations for the ϵ's written down in terms of these values of the α's, and solved for ϵ_1, ϵ_2, and ϵ_3. The phase difference curve corresponding to this first approximation was usually found to fit the observed curve very well excepting at the lower frequencies, where it was occasionally as much as 20 per cent in error. Assuming for ϵ_2 and ϵ_3 the values just found, an equation involving ϵ_1 and α_1 alone may be written down, both for the change of capacity and for tan φ_1. Different values of α_1 are then assumed, the values of ϵ_1 corresponding calculated, and thence φ_1. By interpolation a value of α_1 is quickly obtained which satisfies both curves, and with this value the solution for ϵ_1 ϵ_2, ϵ_3, is repeated. It is in some cases found necessary to slightly change the values of α_2 and α_3 to fit the observations a little more exactly. This method is applicable to the determination of a greater number of constants, so that the calculated curve may pass through a greater number of observed points. Several curves were thus treated, and values of ϵ_1, ϵ_2, ϵ_3, . . . ϵ_6 and α_1, α_2, α_3, ... α_6 obtained. No further light on the behavior of the condensers was, however, thus obtained, and the difficulties of the solution of the problem are much increased.

(b) CALCULATED RESULTS

In Table IV are given the results of the calculation of the constants of a number of the condenser curves.

In the third and fourth columns of the table are given the values of the constants α and ϵ for the various condensers. In the fifth column are the values of the phase difference, calculated therefrom, and in the sixth column the values observed. The differences between the observed and computed values, given in the seventh column, are roughly proportional to the frequency, and

TABLE IV

Condenser	Temp.	a	ϵ	φ calc.	φ obs.	Diff.	Freq.	$\frac{\Delta C}{C}$ calc
	°			″	″	″		
S_3	15	132	0.00575	695	720	28	$33\frac{1}{3}$	0.00784
		770	.00171	627	690	63	100	
		2660	.00463	656	850	194	300	
				444	830	386	900	.00087
S_3	35	42	.02925	1366	1380	14	$33\frac{1}{3}$	.00628
		487	.00186	773	1015	242	100	
		2310	.00361	592	835	243	300	
				338	756	418	900	.00052
Mp	15	150	.00268	364	364	0	$33\frac{1}{3}$	.00431
		1090	.00256	384	384	0	100	
		3260	.00096	357	450	94	300	
				198	450	252	900	.00033
Mp	25	187	.00285	380	380	0	$33\frac{1}{3}$	.00500
		674	.00098	368	384	16	100	
		3260	.00284	376	403	27	300	
				298	426	128	900	.00073
S_{10}	15	209	.00369	541	555	14	$33\frac{1}{3}$	.00911
		888	.00185	619	656	37	100	
		3260	.00553	725	972	247	300	
				582	1084	452	900	.00143
S_{10}	35	45	.00935	578	588	10	$33\frac{1}{3}$	.00545
		562	.00188	475	534	59	100	
		3260	.00378	489	590	101	300	
				390	677	287	900	.00096
B	20	94	.00105	103	105	2	$33\frac{1}{3}$	.00084
		888	.00039	80	80	0	100	
		3260	.00029	68	63	−5	300	
				42	44	2	900	008.00
				° ′	° ′	′		
t_{17}	15	102	.1768	4 28	4 30	2	$33\frac{1}{3}$	.0926
		628	.0274	2 34	3 00	26	100	
		2660	.0341	1 50	2 32	42	300	
				1 04	2 03	59	900	.0065
t_{17}	35	98	.449	12 7	12 12	5	$33\frac{1}{3}$	.2682
		445	.1344	7 28	7 42	14	100	
		1884	.0780	4 36	5 04	28	300	
				2 05	3 41	96	900	.0087

the values of the internal resistance calculated from the differences are in fair agreement, in the majority of the cases, with those obtained by the other method above. Whereas, in that instance, it was assumed that the curves of phase difference should fall with the increasing frequency, these calculations show that this assumption is subject to some modification with Mp and S_{10}, and the observed curves show apparent anomalies which are in agreement with the calculated form of curve.

In the ninth column are calculated the differences between the observed capacity and the geometrical capacity in terms of the latter. These are found by substituting the calculated values of the α's and ϵ's in the equation (32) for $\frac{\Delta C}{C}$. It may be objected that this extrapolated value is open to doubt. It is, however, true in every instance that the capacity falls only slowly above 1000 cycles, and Curtis[22] has shown that the capacity extrapolated over the range of periods between 0.001 second and zero is in good agreement with the value extrapolated from the curve of capacity with different discharge periods and with steady current.

It is, further, a fact that the geometrical capacity thus derived is not very largely dependent on quite considerable errors in the constants α_2 and α_3, on which it rests. It seems, therefore, probable that the value of the geometrical capacity thus calculated is at least a fair approximation to the correct value, and in lieu of accurate methods for the direct determination of this important quantity, this method has much to recommend it.

An examination of the relations between the various constants of the condensers enables us to draw some important conclusions regarding the observed curves in the light of the modified Pellat theory.

(c) DISCUSSION OF OBSERVATIONS IN THE LIGHT OF THIS THEORY

The condenser t_{17} is, we have seen, typical of a class of condensers which includes Cp, Lp, $Be_{1,5}$, $Be_{34,}$, and t_3. In all of these molecules are present whose time constants $\frac{1}{\alpha}$ cover a wide range of

[22] This Bulletin, 6, p. 482; 1910.

values. Although the number present is large with both large and small values of α, those with strong damping (large time constant) predominate. As the temperature rises the number of aperiodically damped molecules of all degrees of damping increases rapidly, but at a greater rate in the case of the small values of α (strong damping) than for the more feebly damped. As a consequence the phase difference rises with the temperature, and most rapidly at the lower frequencies.

The behavior of the better condensers is not simple, although it is satisfactorily explained by the modified Pellat theory. With *Mp*, for example, the number of aperiodically damped molecules is small at the observed temperatures. At 15° the number is largest with small and medium values of α. As the temperature rises to 25° the feebly damped molecules increase more rapidly in number than those with small values of α. The sum total of aperiodically damped molecules is not, however, very different at the two temperatures, which gives an explanation of the fact that the phase difference remains nearly constant over this range of temperature, except at the higher frequencies, where the increase of the number of feebly damped molecules has its greatest effect in changing the phase difference. Another result of this peculiar change in the distribution of the molecules is to give a greater capacity, relatively to the geometrical value, for a frequency of 900 cycles, at 25° than at 15°. Unfortunately the enormous temperature coefficient of the capacity of this condenser in the neighborhood of 35°, taken in connection with the uncertainties in the measurement of the temperature in this region (see p. 509) prevented a trustworthy determination of the constants in the case of the curve at 35°.

In the case of S_{10} at 15° the number of molecules of small time constant are actually more numerous than those of large time constant, and the phase difference increases with increasing frequency until 300 cycles is reached, after which it begins to fall off as in the usual run of condensers. As the temperature is raised, however, the number of strongly damped molecules rapidly increases, while at the same time those with small damping become less numerous, so that at 35° we have the nearly normal

curve of phase difference, except that its fall with the frequency is a little retarded in the region of from 100 to 300 cycles. The capacity of this condenser relatively to the geometrical capacity is unlike that of $M\phi$, larger at 15° and a frequency of 900 per second than at 25° and the same frequency.

The facts are with S_3 very much the same as in the preceding case, except that at 15° the feebly damped molecules, although numerous, are overbalanced by those which are strongly damped. At 35° the number of feebly damped molecules has appreciably decreased, causing a decrease of capacity at 900 cycles (relatively to the geometrical capacity at the temperature in question) in going from 15° to 35°. The phase difference curve at 35° is of quite the usual form.

The constants of the mica condenser B are appended for comparison. The calculated curve of phase difference fits the observed very exactly over the whole range of frequency, indicating that the internal resistance of the condenser taken in connection with its small capacity (0.1 mf) is not large enough to appreciably affect the measured values of the phase difference.

From what has gone before, we see, therefore, that all the peculiarities of the observed curves may be accounted for by the theory with the aid of quite natural assumptions. The nature of the phenomena taking place in an imperfect dielectric are summarized thus:

In a perfect dielectric the molecules (by which name we will designate complexes of ions in general) which have very high frequencies of vibration and small damping are in continual vibration, but their equilibrium positions follow the relatively slow changes of the applied electric field without sensible retardation.

If, due to some cause, molecules be present whose motion is aperiodically damped, the displacements of these anomalous molecules will lag in phase behind the changes of the field, giving rise to a loss of energy in the dielectric, and an increase in the effective value of the capacity of the condenser. The dielectric is to be regarded as imperfect.

In a nearly perfect dielectric the number of these aperiodically damped molecules will be small, and their damping will be feeble.

The distribution of these anomalous molecules as a function of the damping will vary with different condensers. In general it may be said that the greater the proportion of the molecules of feeble damping the more important the absorption effects at high frequency, the effects of the molecules of strong damping being more pronounced at the lower frequencies.

We have thus the case of relatively good condensers—few anomalous molecules, with relatively small damping. In the mica condenser *B* these molecules are distributed over a large range of damping, those of strong damping being predominant, but the number present of all degrees of damping is small. The condition of affairs with the condenser *Mp* at the lower temperature is much the same, only the number of anomalous molecules present is larger than with the mica condenser. With S_3 and S_{10} the proportion of feebly damped molecules is larger, as is also the total number present of all degrees of damping.

As the temperature rises the tendency in all these better paper condensers is for the proportion of the feebly damped molecules to increase to a maximum while the sum total changes but slowly. (See *Mp*.) When, however, this maximum has been reached, the number of strongly damped molecules becomes increasingly important. (See S_3 and S_{10}.)

With increasing temperature a point is at length reached where the total number of aperiodically damped molecules is very appreciable, those of strong damping being present in the largest proportions, and not only does the total number increase as the temperature rises still further, but the relative increase is larger with the strongly damped than with the feebly damped molecules. This last phase, which is illustrated by the better condensers at relatively high temperatures, is characteristic of the poorer condensers at room temperatures. There is experimental evidence to lead to the belief that at sufficiently low temperatures these latter condensers would also behave like the condensers *Mp*, S_3, S_{10}, etc.

5. HOPKINSON'S THEORY OF ELASTIC AFTER EFFECT

In 1876 Hopkinson advanced the theory that the phenomena of absorption owe their origin to strains in the medium which

persist after the removal of the cause which gives rise to them. This idea, which was suggested to him by Maxwell, Hopkinson treats in a manner similar to that adopted by Boltzmann [23] in 1874 in the developement of the theory of the after effects of mechanical strain.

If θ_t be the angle of torsion of a wire at the time t, the couple acting is not only proportional to this angle but previous strains have also an effect. The effect of an angle of torsion $\theta_{t-\omega}$ at a time ω seconds previous will depend on the angle of torsion, the length of time during which it persisted, $d\omega$, and decays with the time in the manner given by some function $f(\omega)$ of the interval which has elapsed up to the time t under consideration.

The torque L is therefore given by the equation

$$L = a\theta_t - \int_o^\infty \theta_{t-\omega} f(\omega) d\omega$$

By analogy Hopkinson wrote, therefore,

$$x_t = y_t - \int_o^\infty y_{t-\omega} \varphi(\omega) d\omega \qquad (40)$$

where x_t = the potential between the terminals at the time t and y_t = the surface integral of the displacement, divided by the geometrical capacity, and he then went on to apply this equation to his measurements with a steady applied electromotive force.

Let us now adapt this hypothesis to the case of alternating currents. In an ideal dielectric the displacement is given by $D = \frac{E_o K}{4\pi} \sin pt$, where E_o is the amplitude of the electric field. The actual displacement in the imperfect dielectric is greater by an amount D', which is the integral of the residual displacements, due to the previous values of the applied emf. At any time τ seconds previous to the time under consideration, the emf. existing at that time would produce an residual effect proportional to its value $E_o \sin p\ (t-\tau)$, and to the time $d\tau$ during which it was applied, and this displacement would fall off according to some

[23] Wien. Ber. II, 70, Oct., 1874.

function of the elapsed time τ. We can write therefore

$$D' = \int_o^\infty \frac{E_o K}{4\pi} \sin p(t-\tau) \cdot f(\tau) d\tau.$$

If we now make the assumption that the residual displacements decay according to a simple exponential law, and set $f(\tau) = \alpha\epsilon e^{-\alpha\tau}$ we find, remembering that

$$\int_o^\infty e^{-\alpha\tau} \sin p(t-\tau) d\tau = \int_{-\infty}^t e^{-\alpha t} e^{\alpha T} \sin pT dT$$

the total displacement D will be given by

$$D = \frac{E_o K}{4\pi} \sin pt + \frac{E_o K}{4\pi} \alpha\epsilon \left[\frac{\alpha \sin pt - p \cos pt}{p^2 + \alpha^2} \right]$$

The current i may be found at once

$$i = \frac{dD}{dt} = \frac{pKE_o}{4\pi} \left[\left(1 + \frac{\alpha^2\epsilon}{p^2 + \alpha^2} \right) \cos pt + \frac{\alpha\epsilon p}{p^2 + \alpha^2} \sin pt \right]$$

from which the capacity C' is given in terms of the geometrical capacity C_o by the relation

$$C' = C_o \left[1 + \frac{\alpha^2\epsilon}{p^2 + \alpha^2} \right]$$

and the phase difference is

$$\tan \varphi = \frac{\alpha\epsilon p}{p^2 + \alpha^2(1+\epsilon)}$$

These are seen to be Pellat's equations, as applied to alternating currents, and if instead we had assumed a steady applied electromotive force we would have found for the displacement

$$D = \frac{E_o K}{4\pi} [1 + \epsilon(1 - e^{-\alpha\tau})]$$

which is Pellat's initial equation (22).

According to this way of looking at the matter, we have that the constant α is analogous to the reciprocal of the coefficient of

viscosity, while ϵ finds its analogy in the reciprocal of Young's modulus.

Hopkinson did not, however, find evidence of such a simple law as the above. We find the following statement in his Scientific Papers, page 2 (Phil. Trans. 166, II, p. 490, 1876):

> The jar was slightly charged and insulated, and the potentials read off from time to time. It was found (1) that even after 24 hours the percentage of loss per hour continued to increase, (2) that the potential could not be expressed as a function of the time by two exponential terms. But the latter fact was more clearly shown by the rate of development of the residual charge after different periods of discharge, which put it beyond doubt that if the potential is properly expressed by a series of exponential terms at all, several such terms will be required.

Later, when developing his theory of elastic after effect, he says: "It should be noted that the view of this subject adopted by the author in a previous paper can be included in equation (2), (here equation (40)), by assuming that $f(\omega)$ is the sum of a series of exponentials."

We see, therefore, that if the most obvious assumptions be made in Hopkinson's theory with regard to the law of decay of the elastic after effects, we are led mathematically to Pellat's theory and von Schweidler's modification of the same.

III. SUMMARY AND CONCLUSION

The principal results of this investigation may be summarized as follows:

1. The capacities and phase differences of 13 commercial paper condensers by different English, French, German, and American makers were measured at temperatures ranging from 10° to 35°, and at frequencies lying between 33 and 1000 cycles per second, using bridge methods previously developed at the Bureau of Standards and only briefly touched upon here.

2. The results of these measurements are presented in 26 curves, which show, for each of the condensers studied, the change of capacity and phase difference with change of temperature, and the variation of the capacity and phase difference with change of frequency, the corresponding curves of a representative mica condenser being appended for sake of comparison. These curves show that the changes of capacity and phase difference with changes of temperature and frequency are much larger than the

corresponding changes with mica condensers. In general, these effects of temperature and frequency are larger the larger the absolute value of the phase difference. They are, however, subject to no simple laws. (For the more detailed discussion of the results of the curves, see p. 514 et seq.). Paper condensers are not suitable to serve as standards of capacity, and should not be employed in *alternating-current* work, where it is desired that the capacity shall remain constant, except when a careful study of the condenser, under different conditions of temperature and frequency, has shown that a satisfactory constancy may be expected. The use of paper condensers for *direct-current* measurements is to be condemned.

3. The phase differences observed lie between 6 minutes and the enormous value of 22°. A phase difference of several degrees is not uncommon in commercial telephone condensers. It is shown that the internal resistance of the plates and leads of a paper condenser are often large, especially in the case of telephone condensers made by rolling up together sheets of tin foil and paper. In one example the energy loss in the condenser, external to the dielectric, is at 1000 cycles three times as great as the energy loss in the dielectric.

4. The observed curves are discussed in the light of the following theories of absorption: Maxwell's theory of a heterogeneous dielectric, Houllevigue's theory of a viscous frictional loss of energy, Pellat's theory, and von Schweidler's modification of it, and Hopkinson's theory of elastic after effects. In each case the expressions for the capacity and phase difference are derived, assuming the wave of emf to be sinusoidal.

5. It is shown that Hopkinson's theory, subject to the assumptions made by its author, is mathematically equivalent to von Schweidler's modified Pellat theory.

6. Of all the theories examined that of von Schweidler only could be made to give quantative results in agreement with the observations. In this theory the difference in phase between the current in an actual condenser and that which would flow if the dielectric were free from absorption is assumed to be due to the presence in the dielectric of molecules whose vibrations are

aperiodically damped. To account for the complicated observed relations connecting the capacity and phase difference with the temperature and frequency it is only necessary to assume a different law of distribution of these molecules as a function of the time constant, for each condenser and for the same condenser at different temperatures. It is shown that the observed results may be represented with a good degree of accuracy on the simple assumption that the actual distribution of the aperiodic molecules is equivalent to one in which molecules of only three different values of time constant are present. The constants of a number of the observed curves have been calculated on this assumption, the method of their calculation being indicated. Such results give a clear idea of the reasons for apparent anomalies in some of the curves in the light of this theory. Perhaps the most important application of such calculations is to the determination of the value of the geometric capacity (the ideal value as unaffected by absorption). This method gives a valuable check on the method of extrapolation described by Curtis (loc. cit., p. 482).

In conclusion it may be stated that further progress in the unraveling of the obscure points remaining for solution in the theory of an imperfect dielectric will probably best be attacked in the future by making accurate measurements of the nature of those in this investigation on condensers constructed under known and carefully controlled conditions. For example, it is important to make measurements on pure, homogeneous substances in the solid or liquid form, to settle the question whether such substances are entirely free from absorption, as has often been maintained.[25] The quantitative effect, on the value of the phase difference, of known added impurities also requires investigation, and such experiments will probably suggest other and equally fruitful points for examination in the light of the dielectric theory.

The author is indebted to Professor Rosa for valuable suggestions in the course of the work, and to Mrs. Grover for assistance in making the observations.

WASHINGTON, February 28, 1911.

[25] Rowland and Nichols, Phil. Mag. (5), **11**, p. 414; 1881. Muraoka, Wied. Ann., **40**, p. 328; 1890. Maclès, Jour. de Phys., **8**, p. 631; 1909.

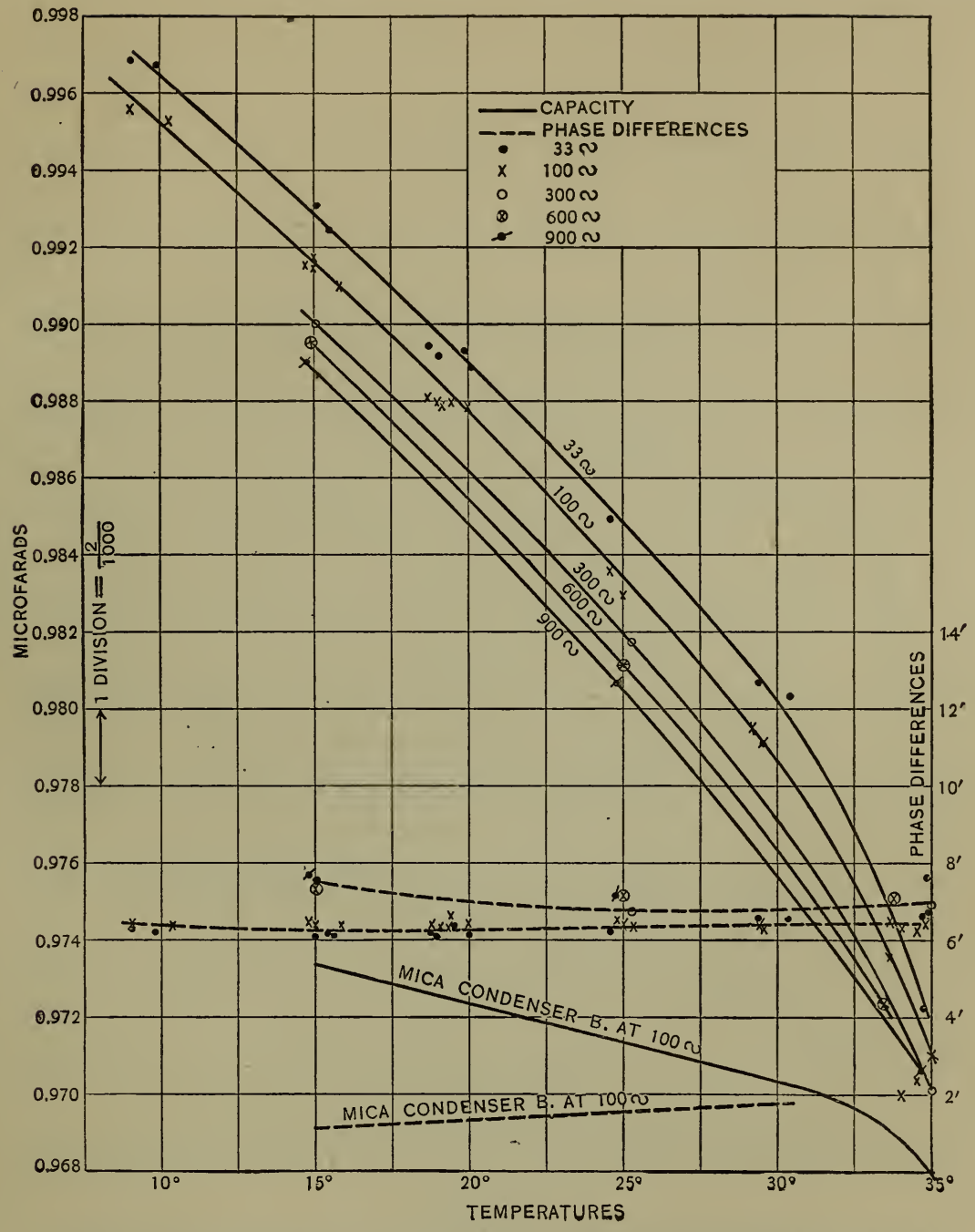

Fig. 5.—*Curves of Capacity and Phase Difference of Mp as Functions of the Temperature*

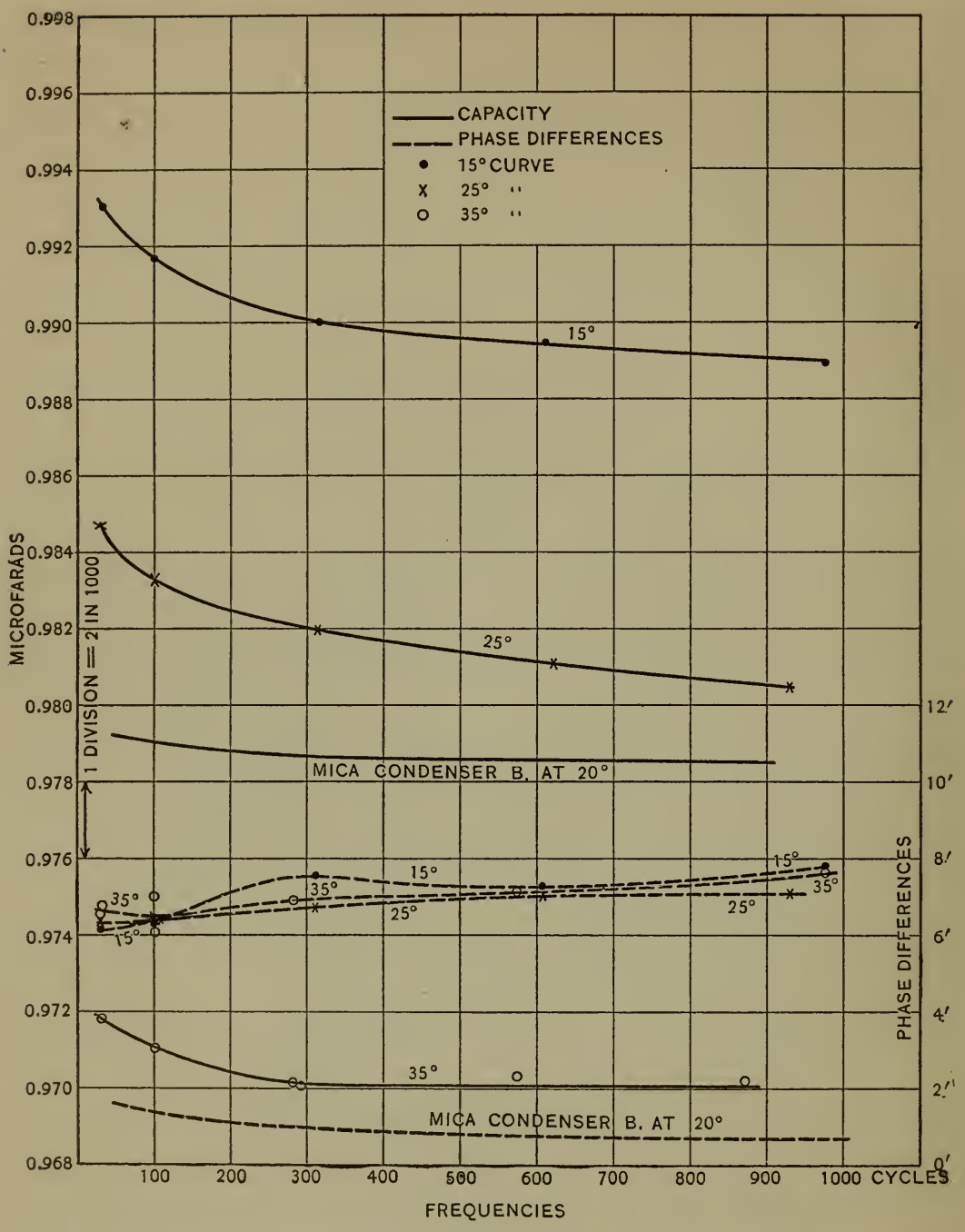

Fig. 6.—*Curves of Capacity and Phase Difference of Mp as Functions of the Frequency*

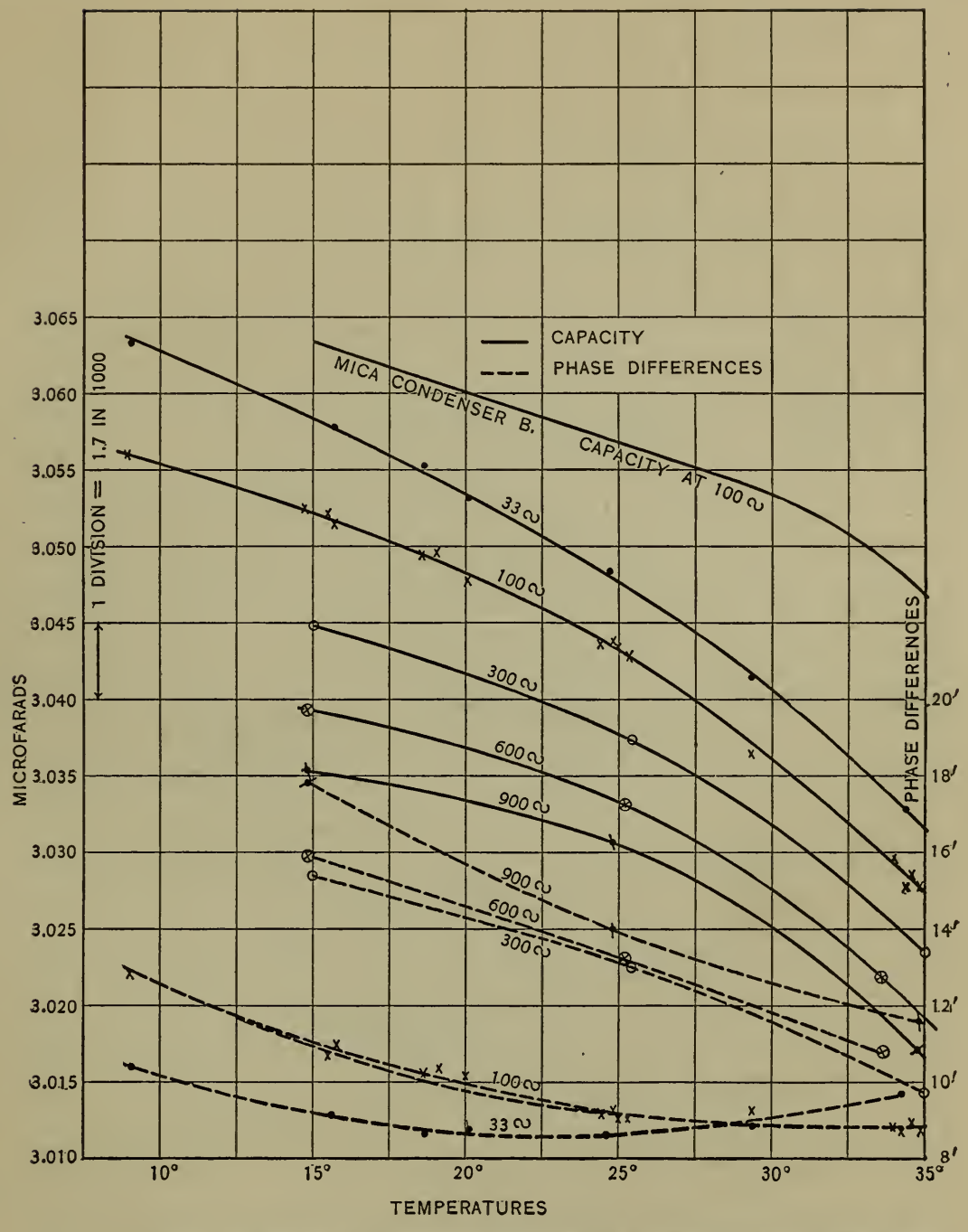

Fig. 7.—*Curves of Capacity and Phase Difference of S_{10} as Functions of the Temperature*

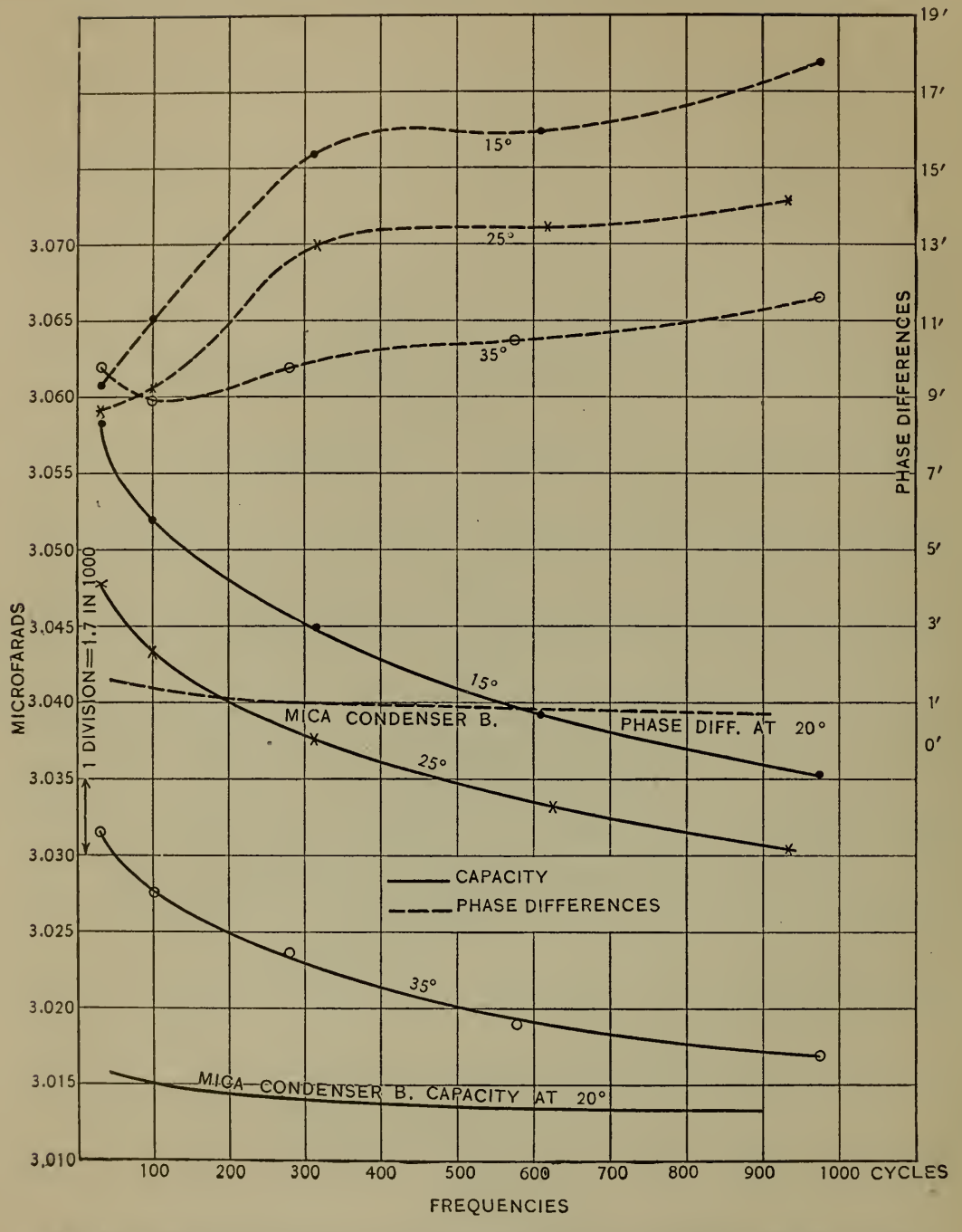

Fig. 8.—*Curves of Capacity and Phase Difference of S_{10} as Functions of the Frequency*

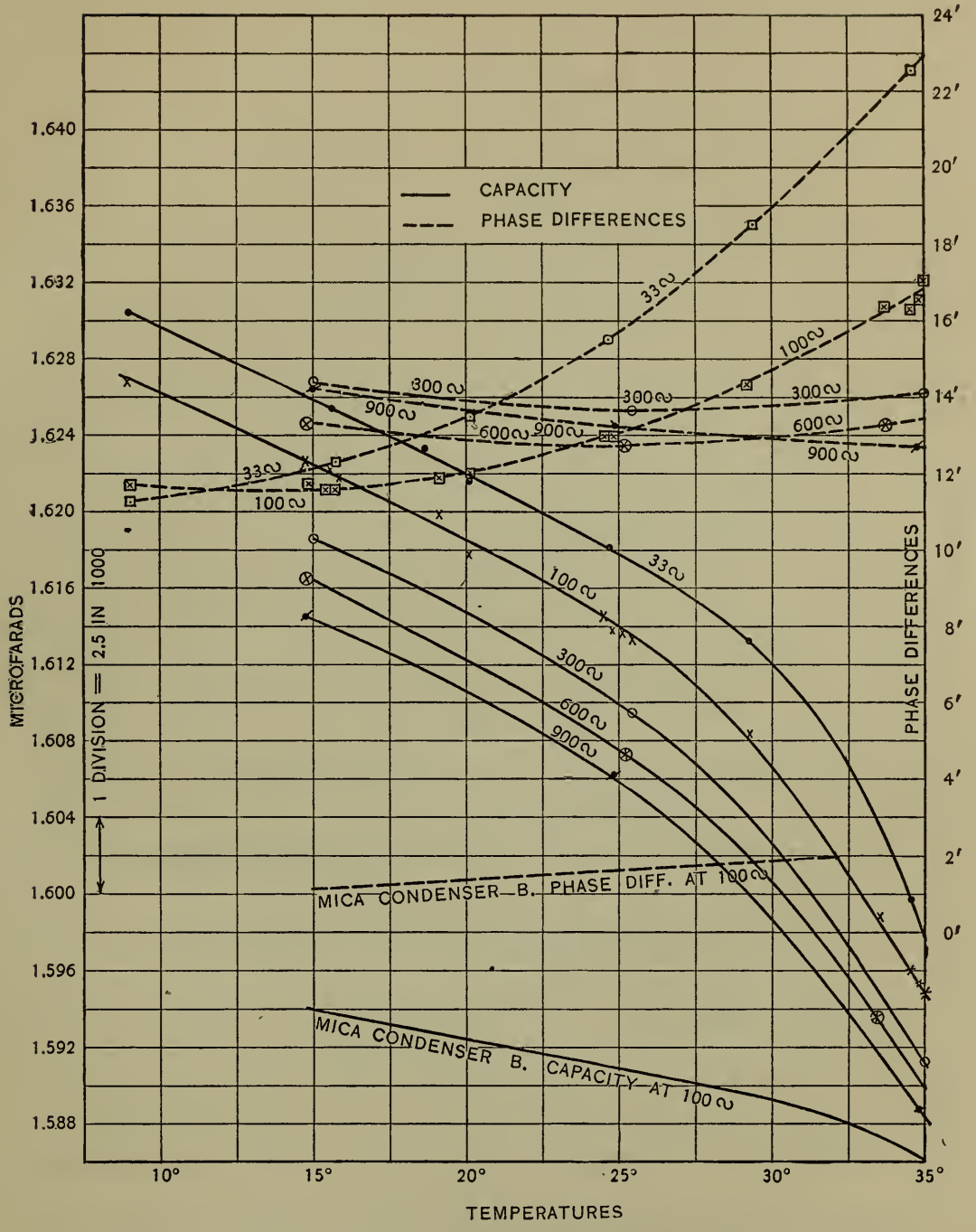

Fig. 9.—*Curves of Capacity and Phase Difference of S_3 as Functions of the Temperature*

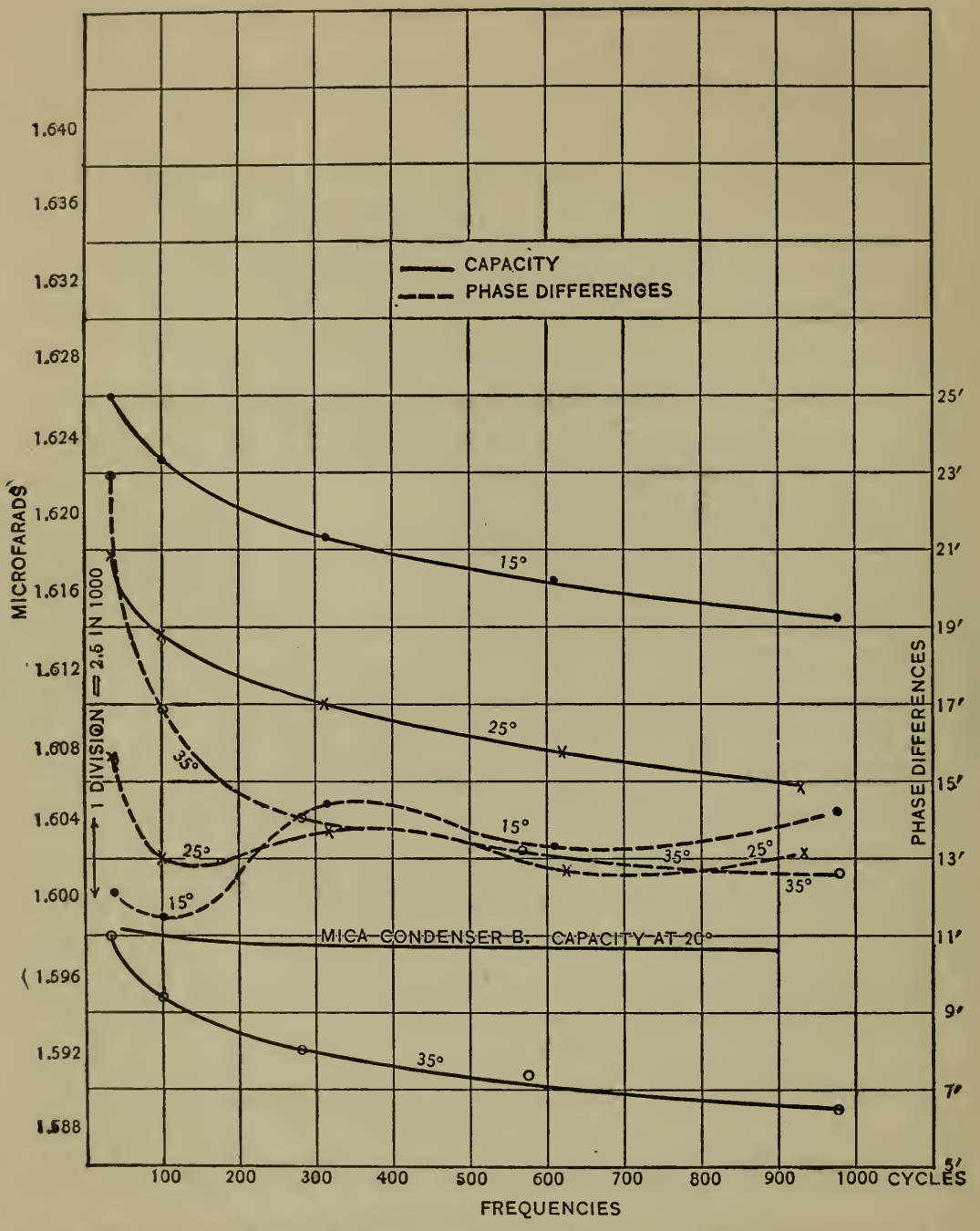

Fig. 10.—*Curves of Capacity and Phase Difference of S_3 as Functions of the Frequency*

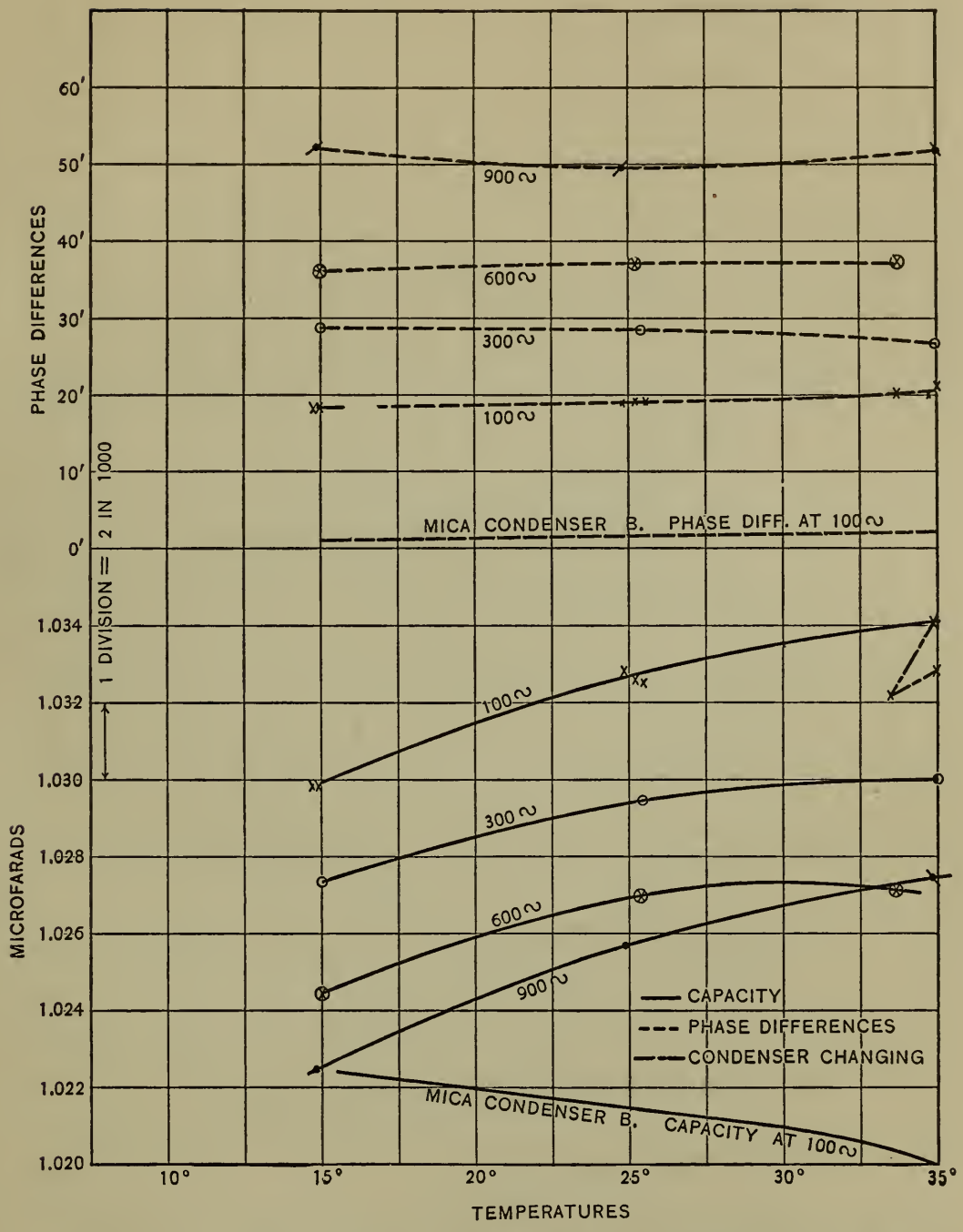

Fig. 11.—*Curves of Capacity and Phase Difference of* W_a *as Functions of the Temperature*

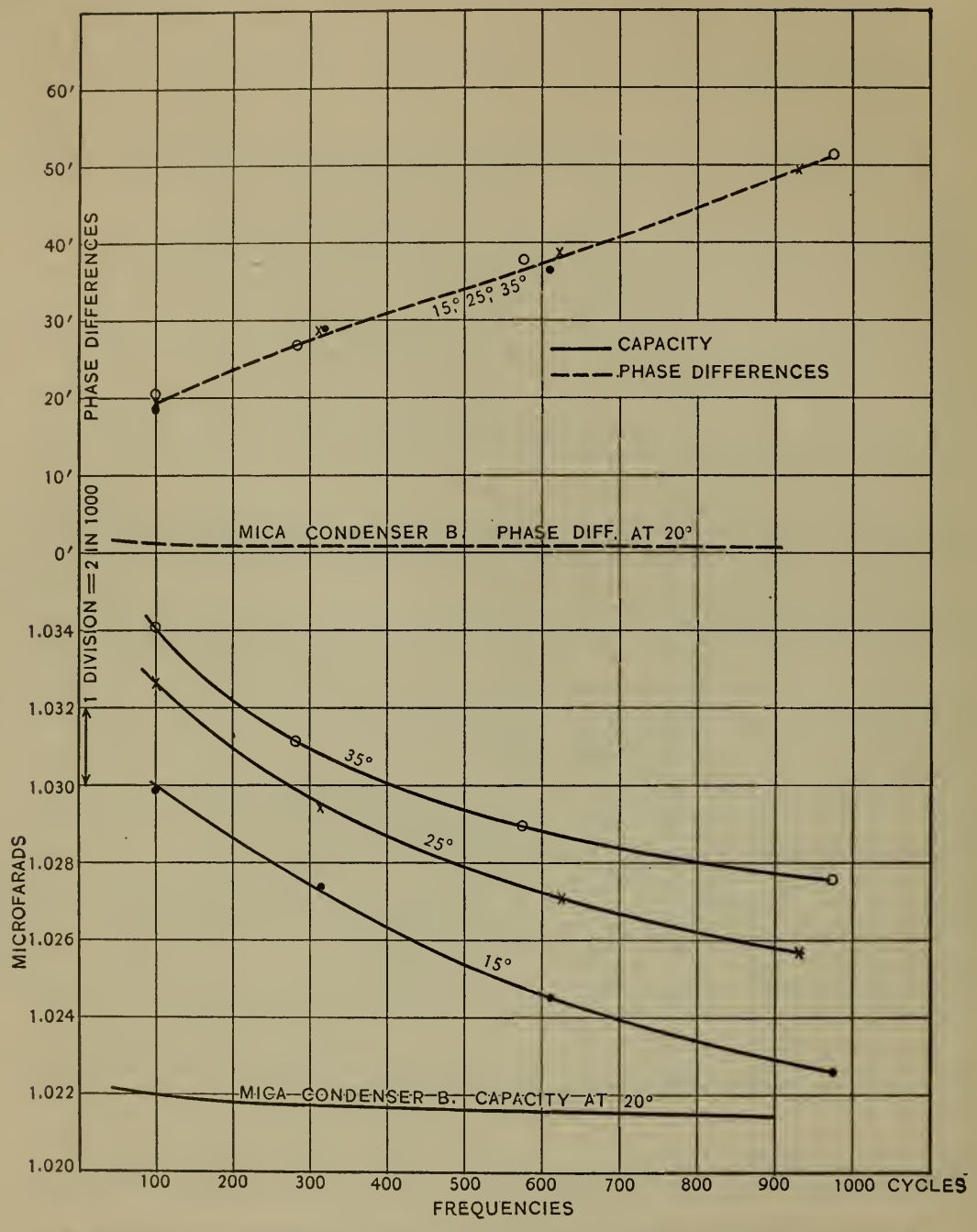

Fig. 12.—*Curves of Capacity and Phase Difference W_a as Functions of the Frequency*

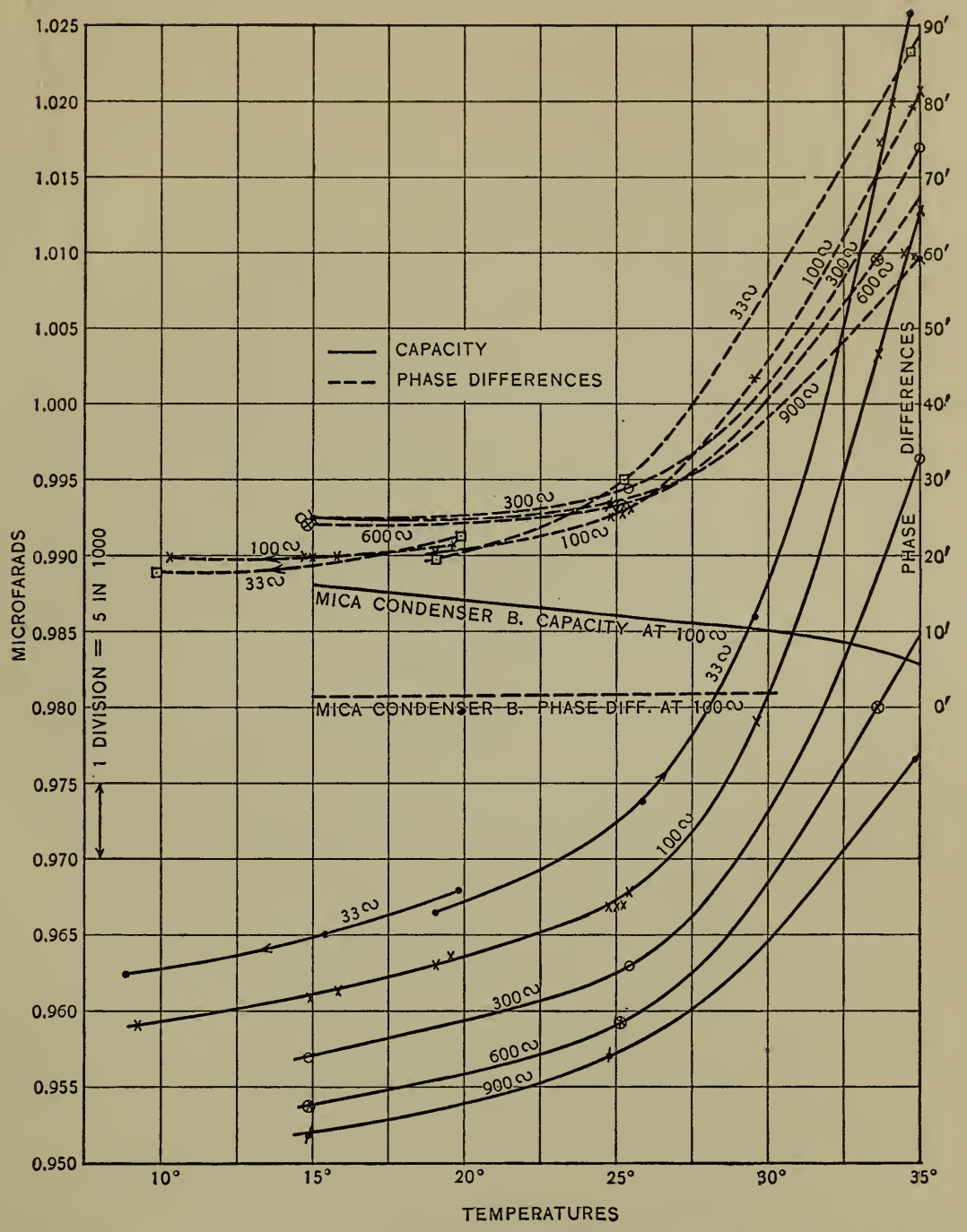

Fig. 13.—*Curves of Capacity and Phase Difference of Lp as Functions of the Temperature*

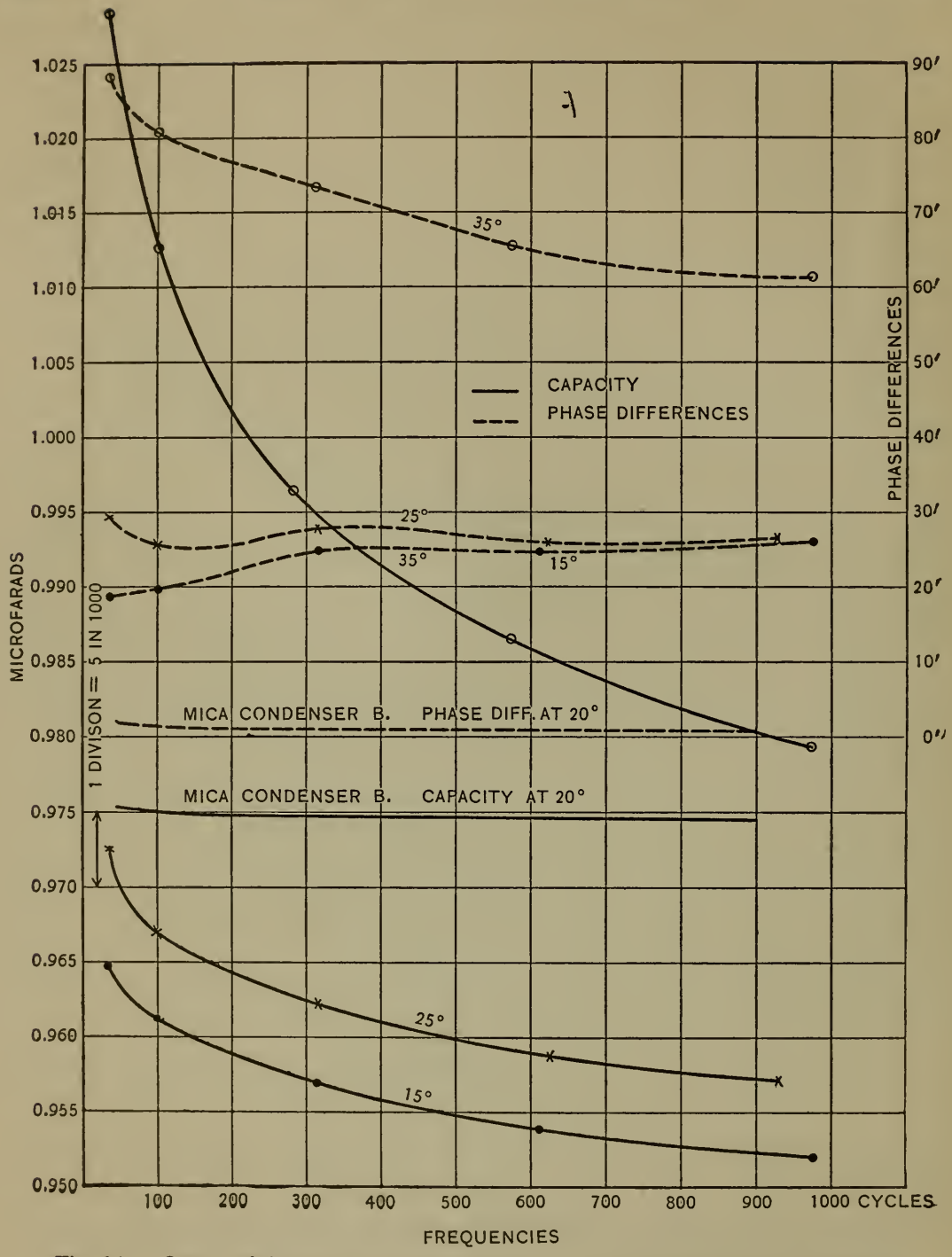

Fig. 14.—*Curves of Capacity and Phase Difference of Lp as Functions of the Frequency*

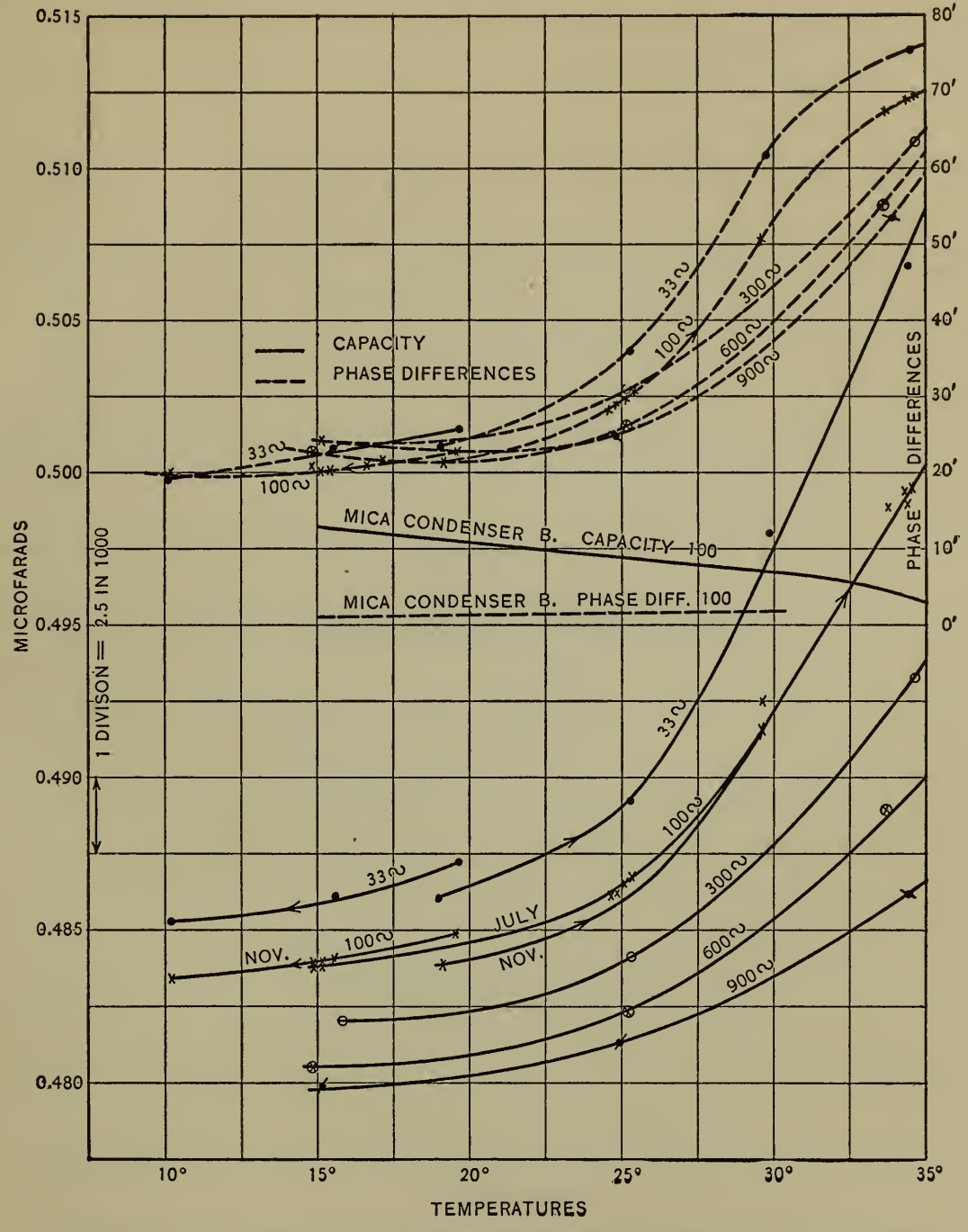

Fig. 15.—*Curves of Capacity and Phase Difference of Cp as Functions of the Temperature*

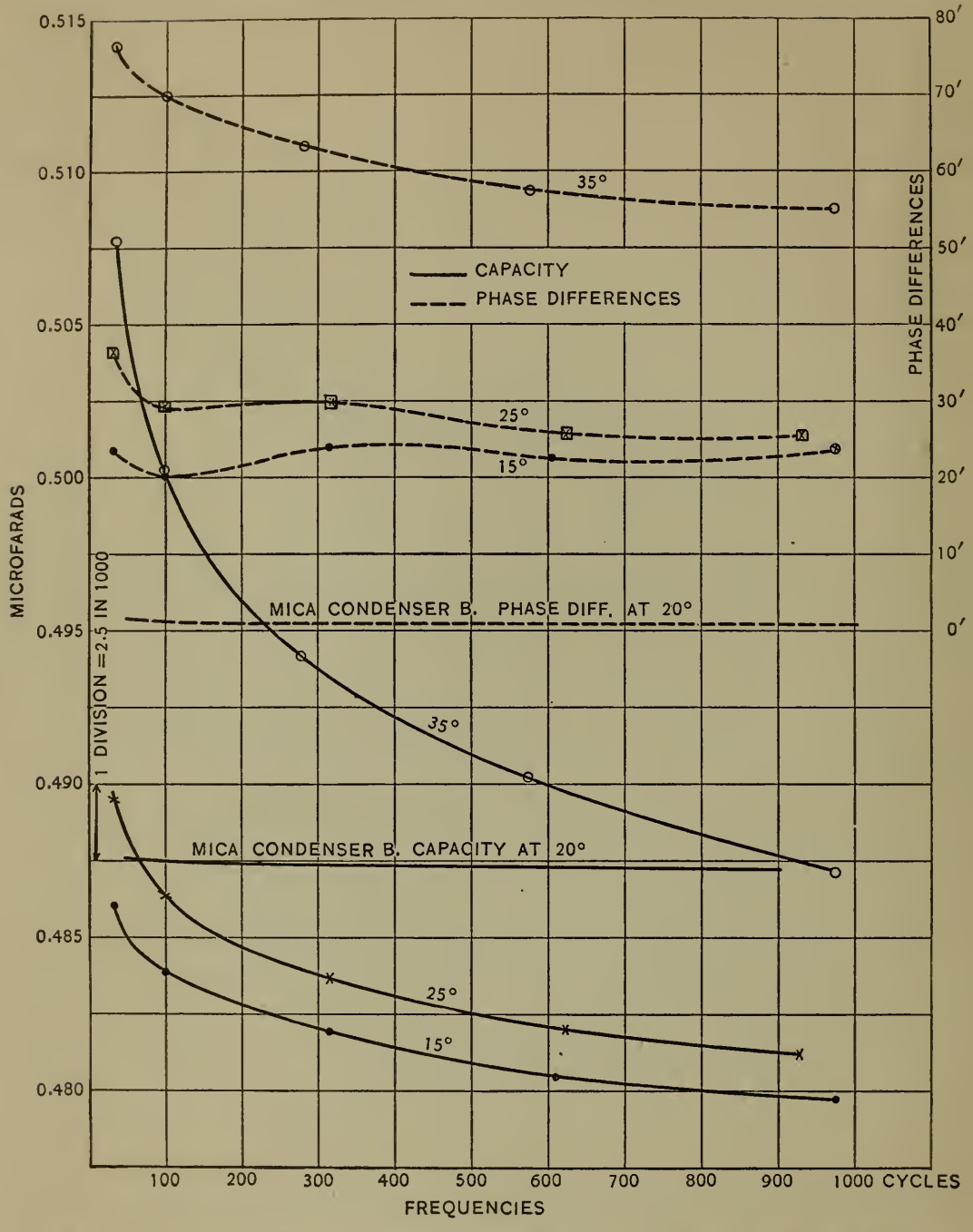

Fig. 16.—*Curves of Capacity and Phase Difference of Cp as Functions of the Frequency*

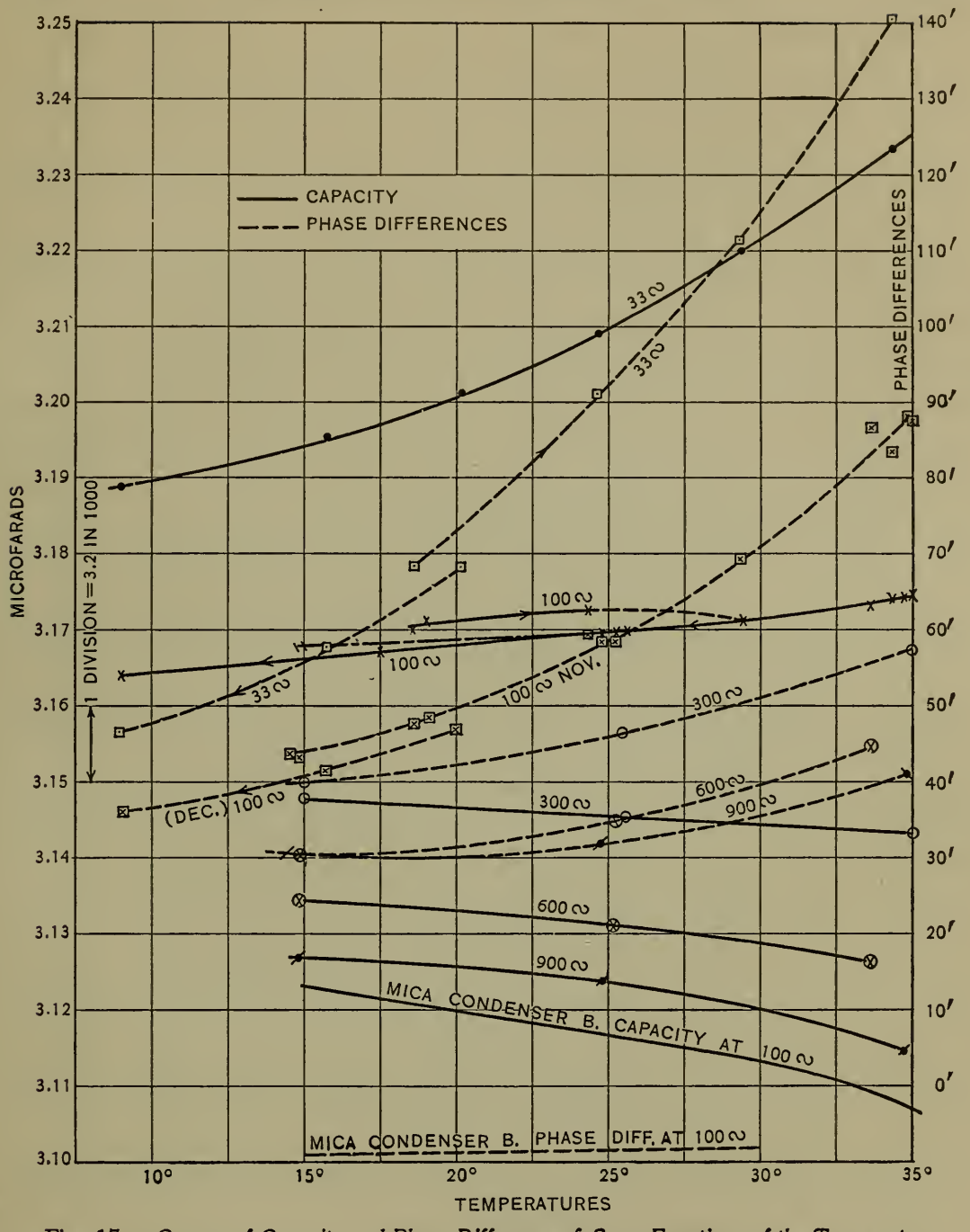

Fig. 17.—*Curves of Capacity and Phase Difference of* S_6 *as Functions of the Temperature*

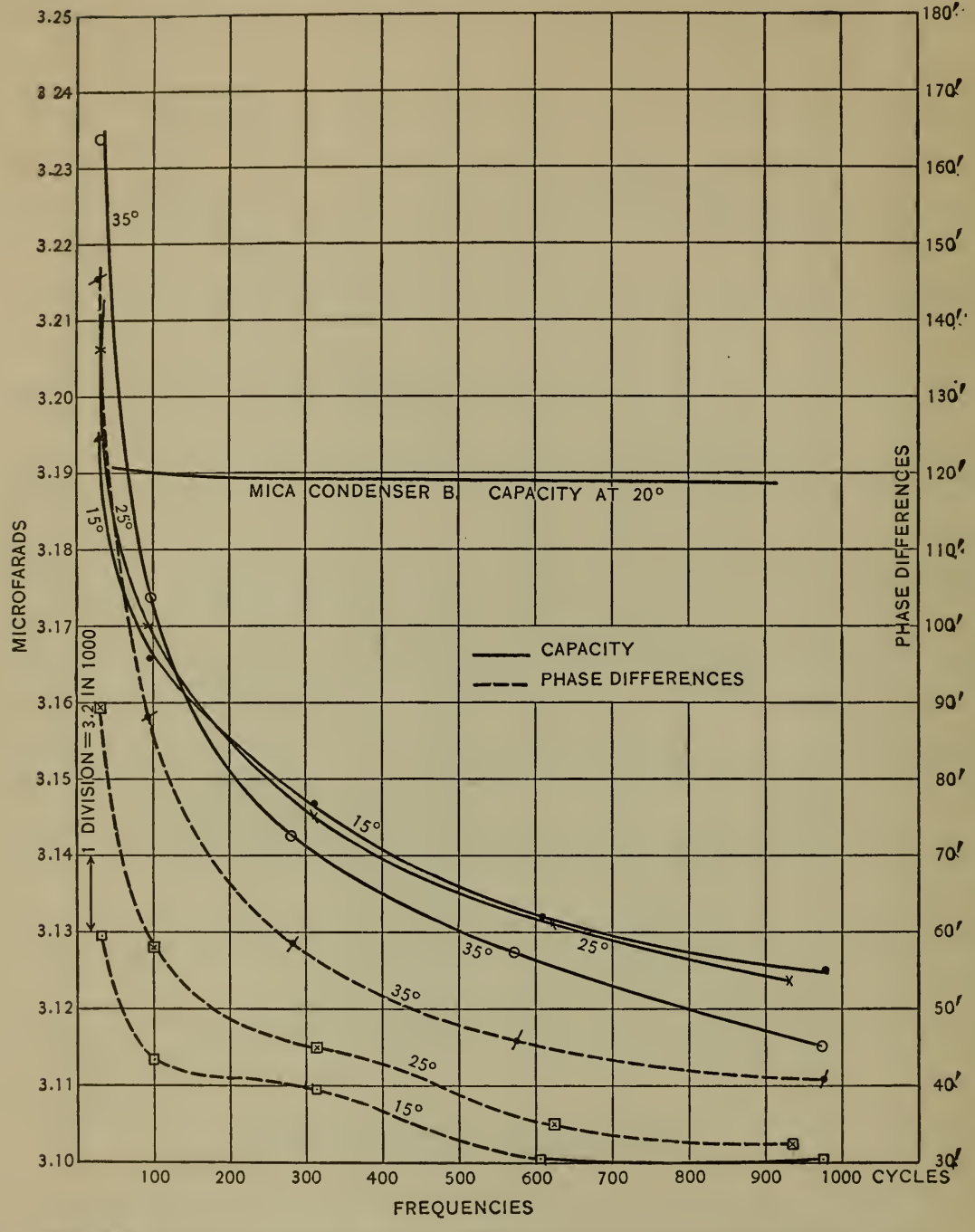

Fig. 18.—*Curves of Capacity and Phase Difference of S_6 as Functions of the Frequency*

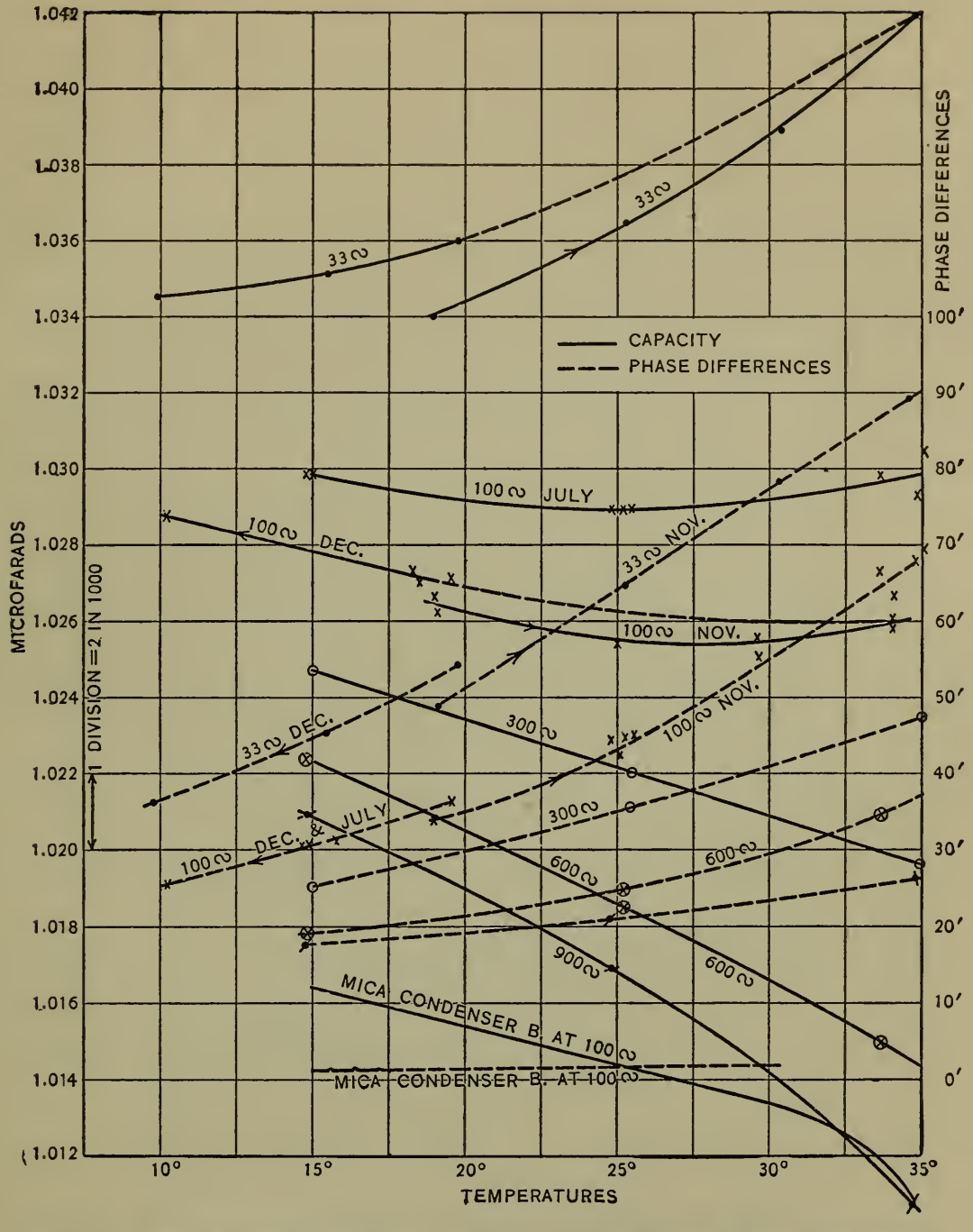

Fig. 19.—*Curves of Capacity and Phase Difference of SH as Functions of the Temperature*

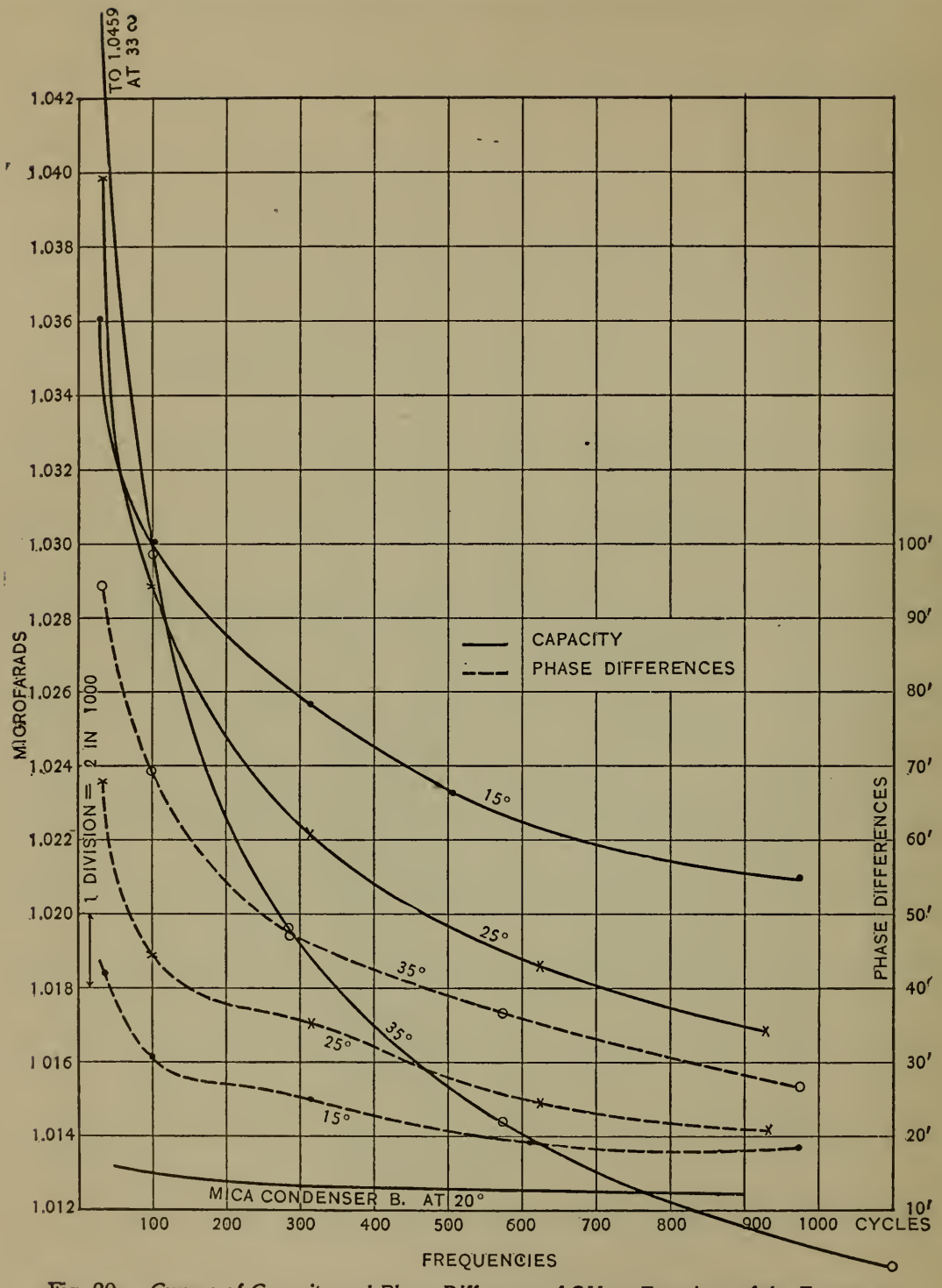

Fig. 20.—*Curves of Capacity and Phase Difference of SH as Functions of the Frequency*

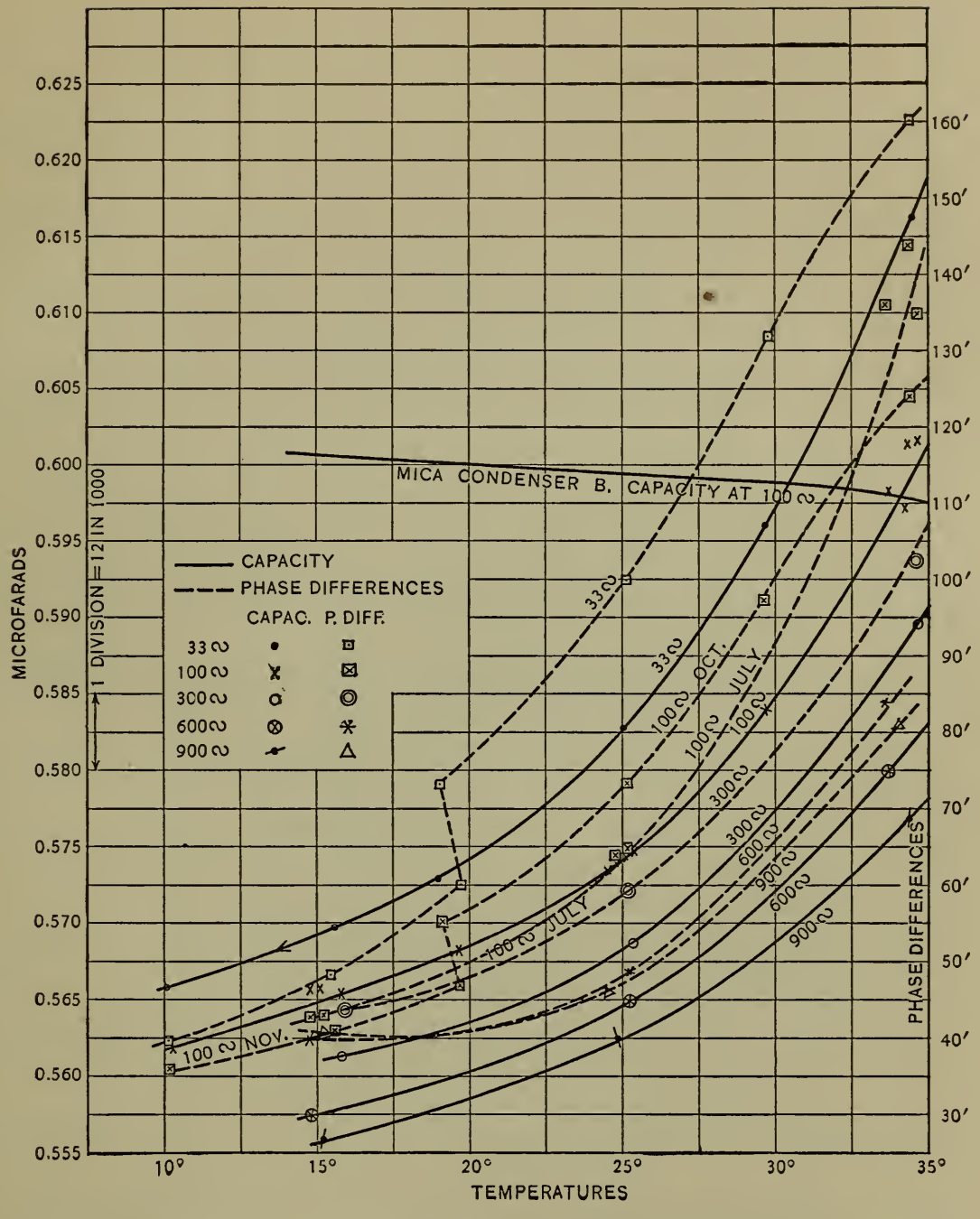

Fig. 21.—*Curves of Capacity and Phase Difference of* $Be_{1,5}$ *as Functions of the Temperature*

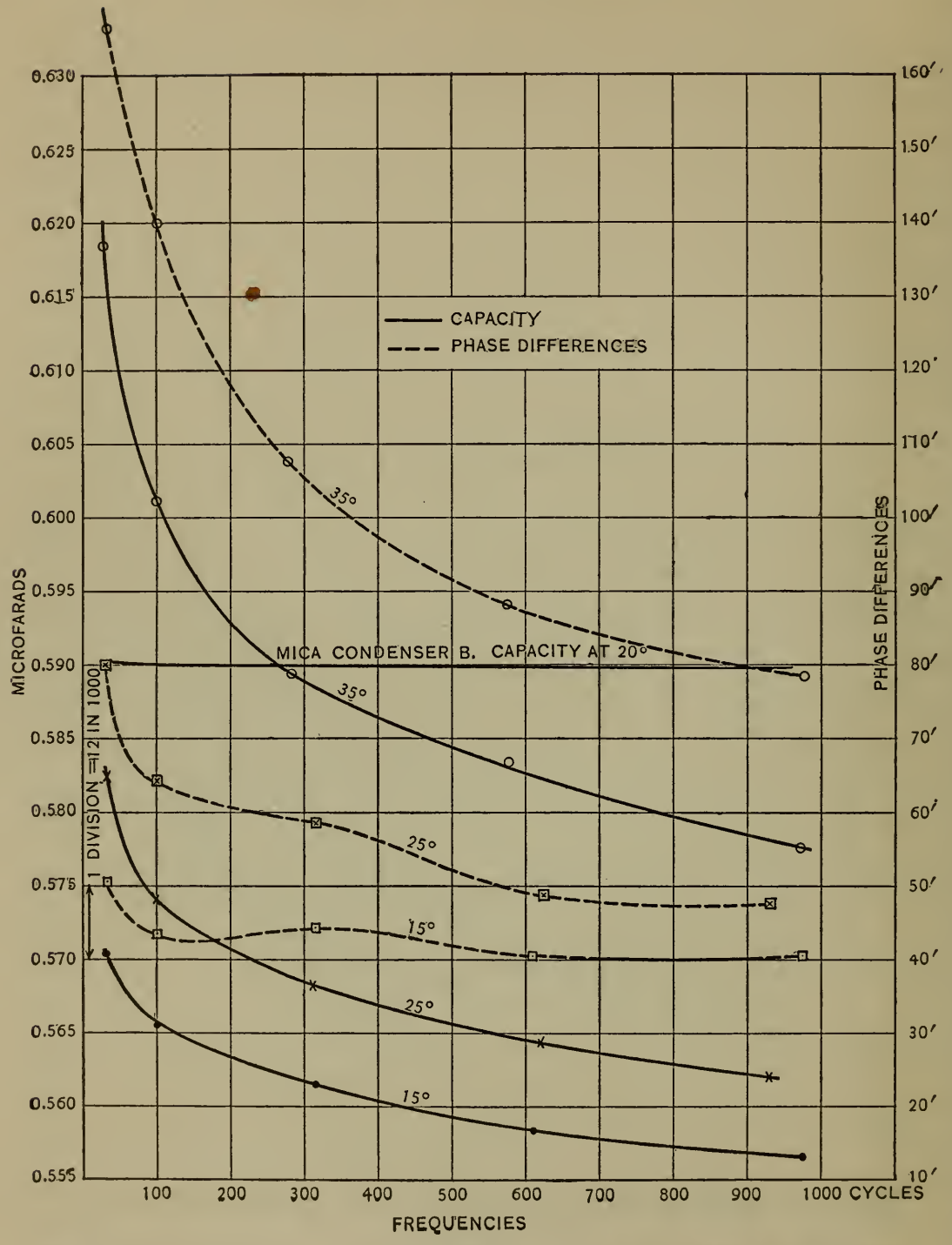

Fig. 22.—*Curves of Capacity and Phase Difference of* $Be_{1,5}$ *as Functions of the Frequency*

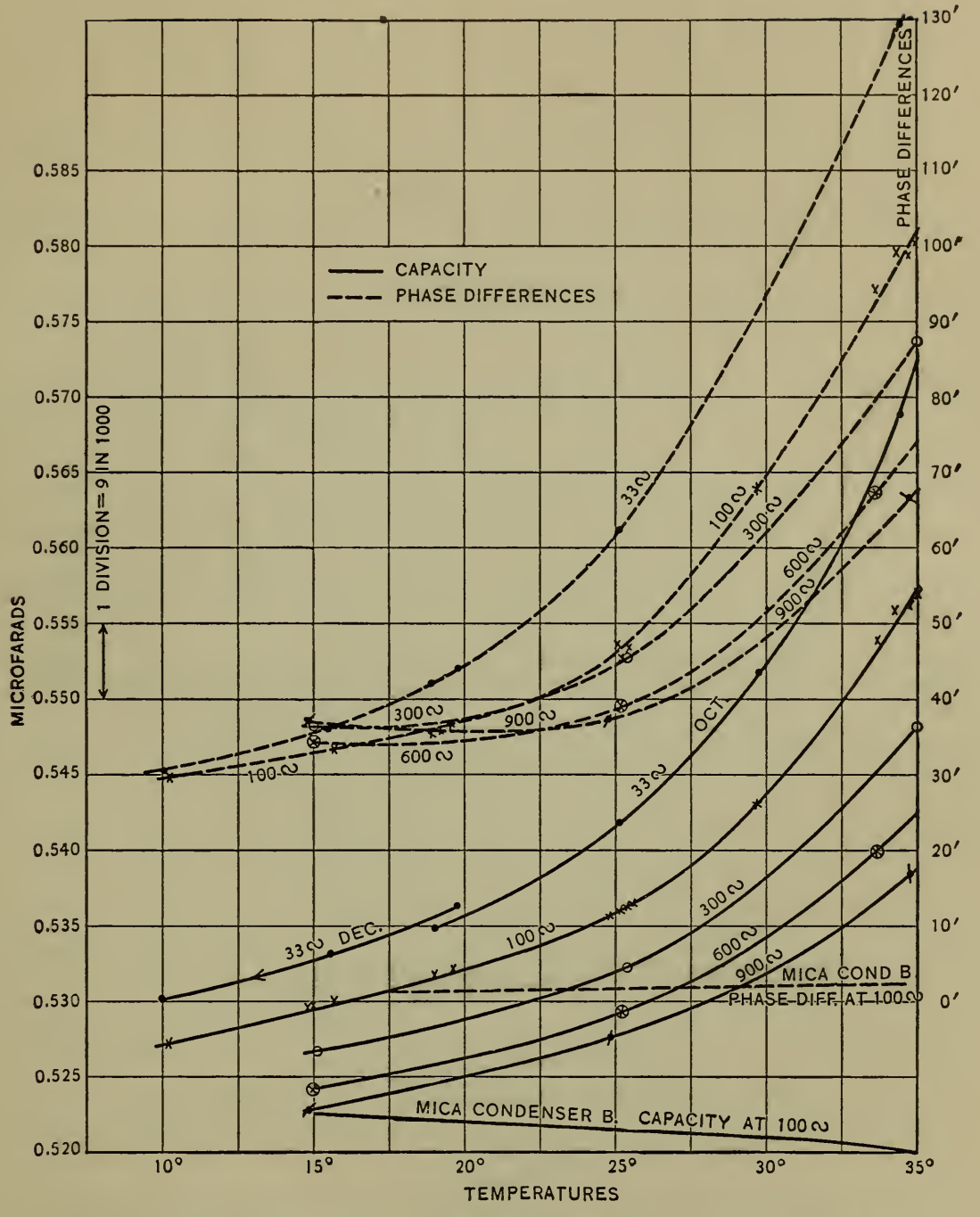

Fig. 23.—*Curves of Capacity and Phase Difference of* $Be_{3,4}$ *as Functions of the Temperature*

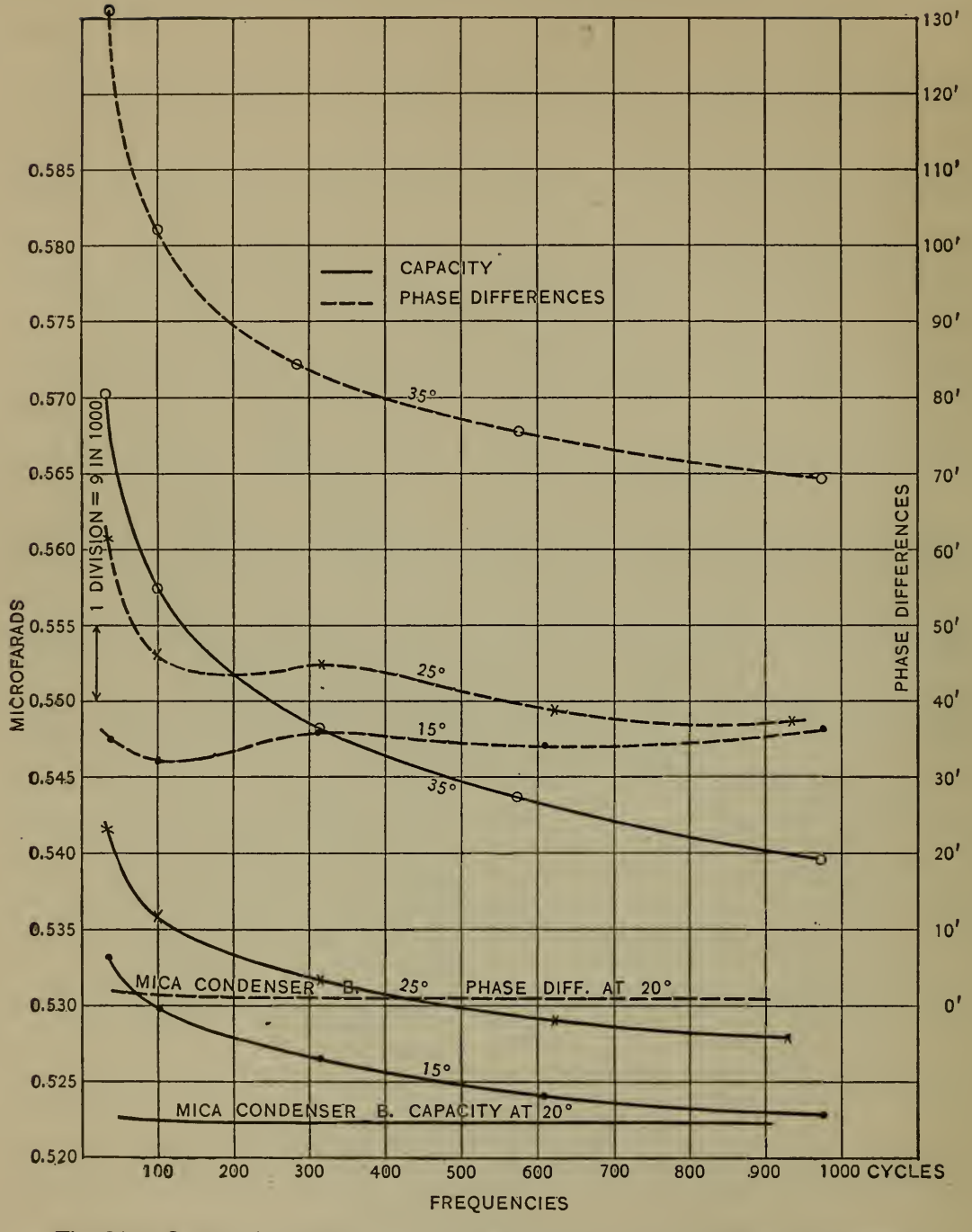

Fig. 24.—*Curves of Capacity and Phase Difference of* $Be_{3,4}$ *as Functions of Frequency*

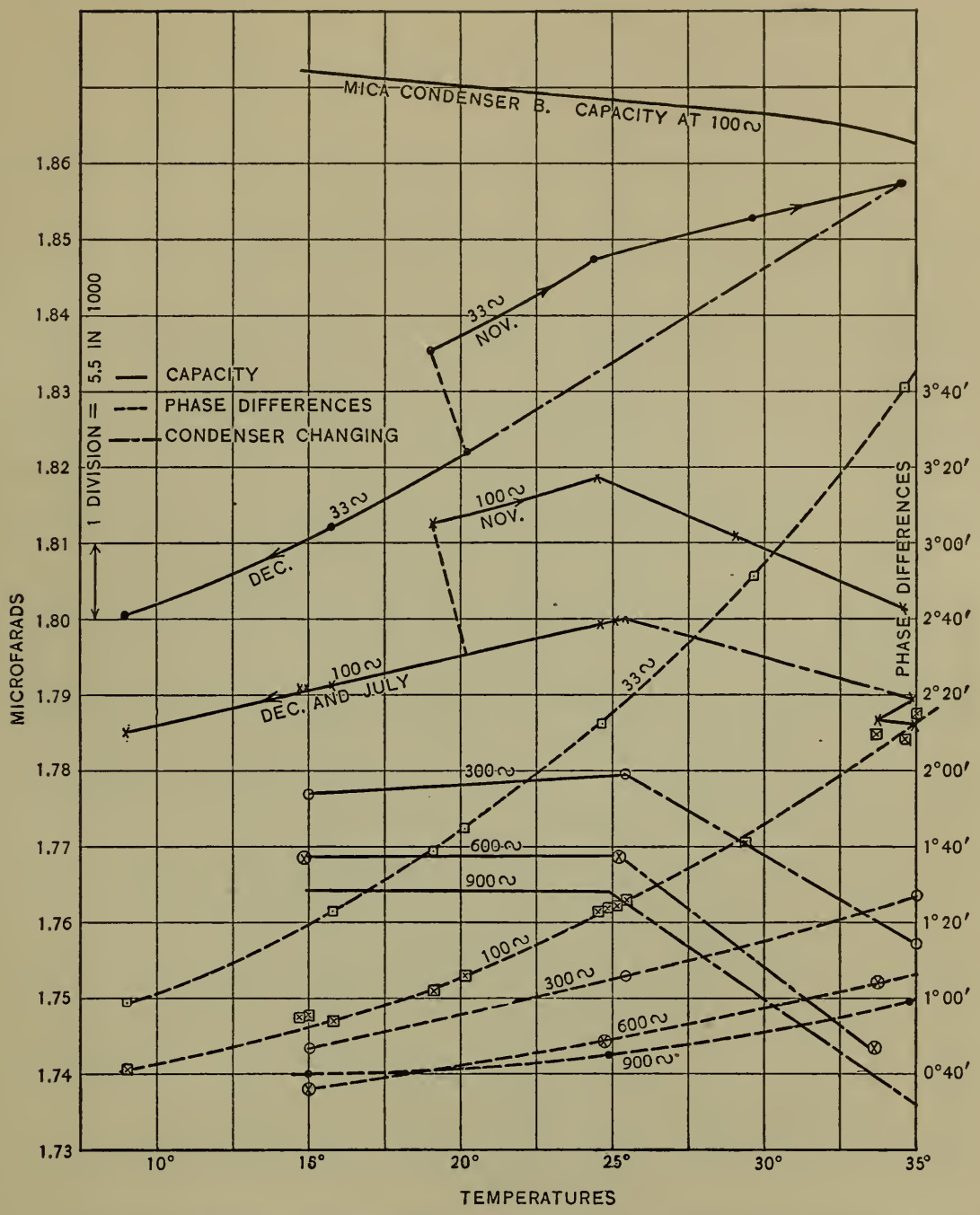

Fig. 25.—*Curves of Capacity and Phase Difference of* T_2 *as Functions of the Temperature*

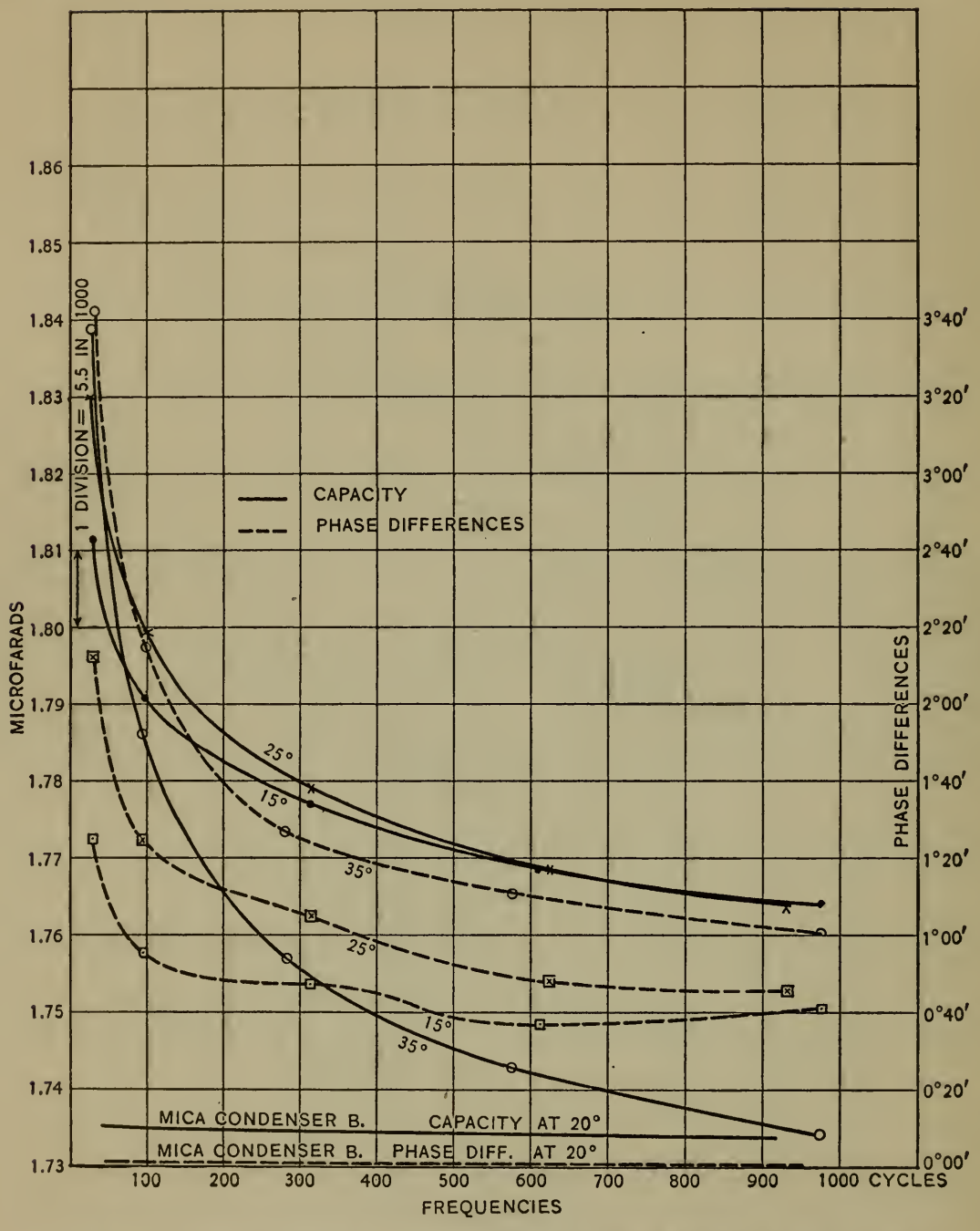

Fig. 26.—*Curves of Capacity and Phase Difference of T_2 as Functions of the Frequency*

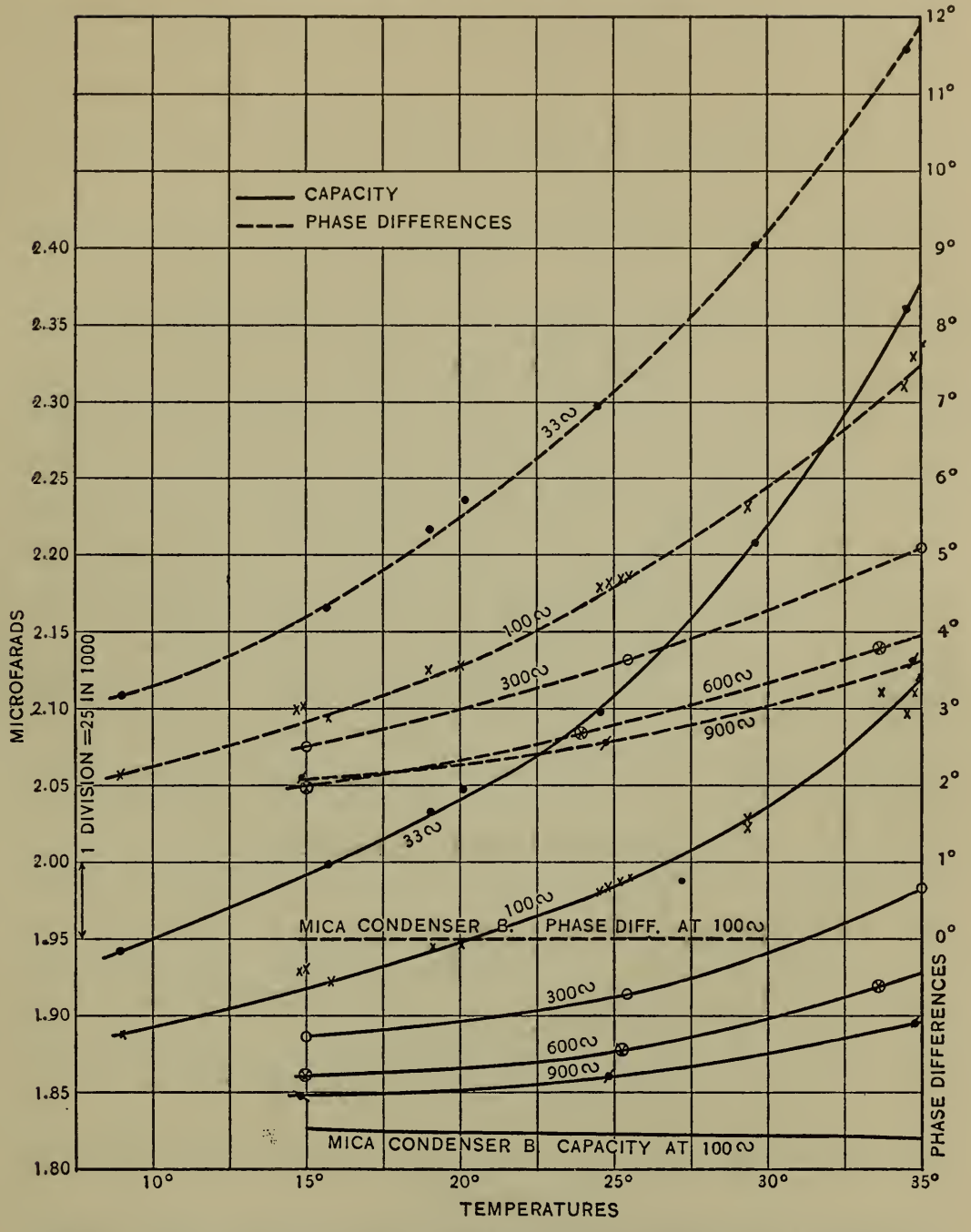

Fig. 27 —*Curves of Capacity and Phase Difference of* t_{17} *as Functions of the Temperature*

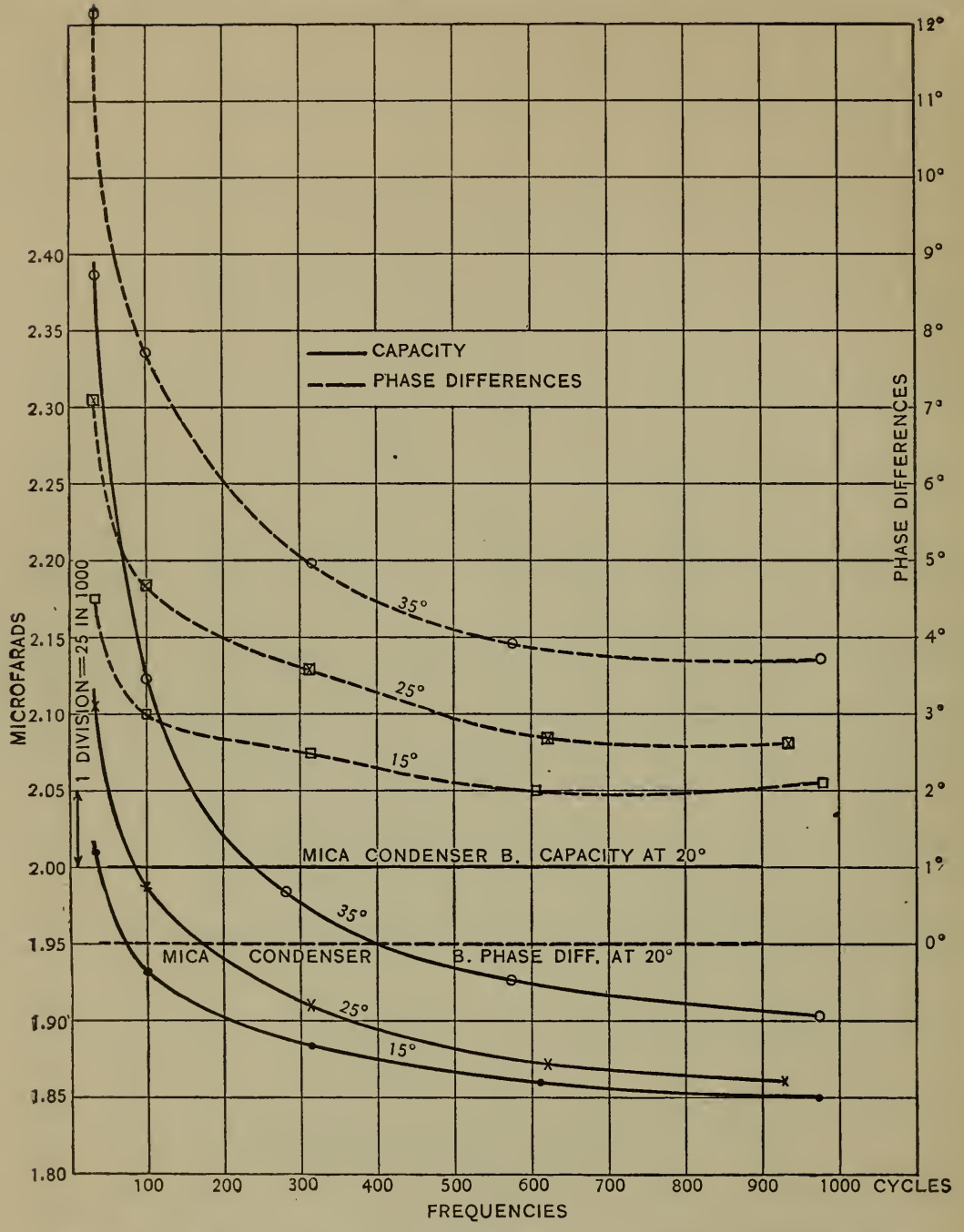

Fig. 28.—*Curves of Capacity and Phase Difference of* t_{17} *as Functions of the Frequency*

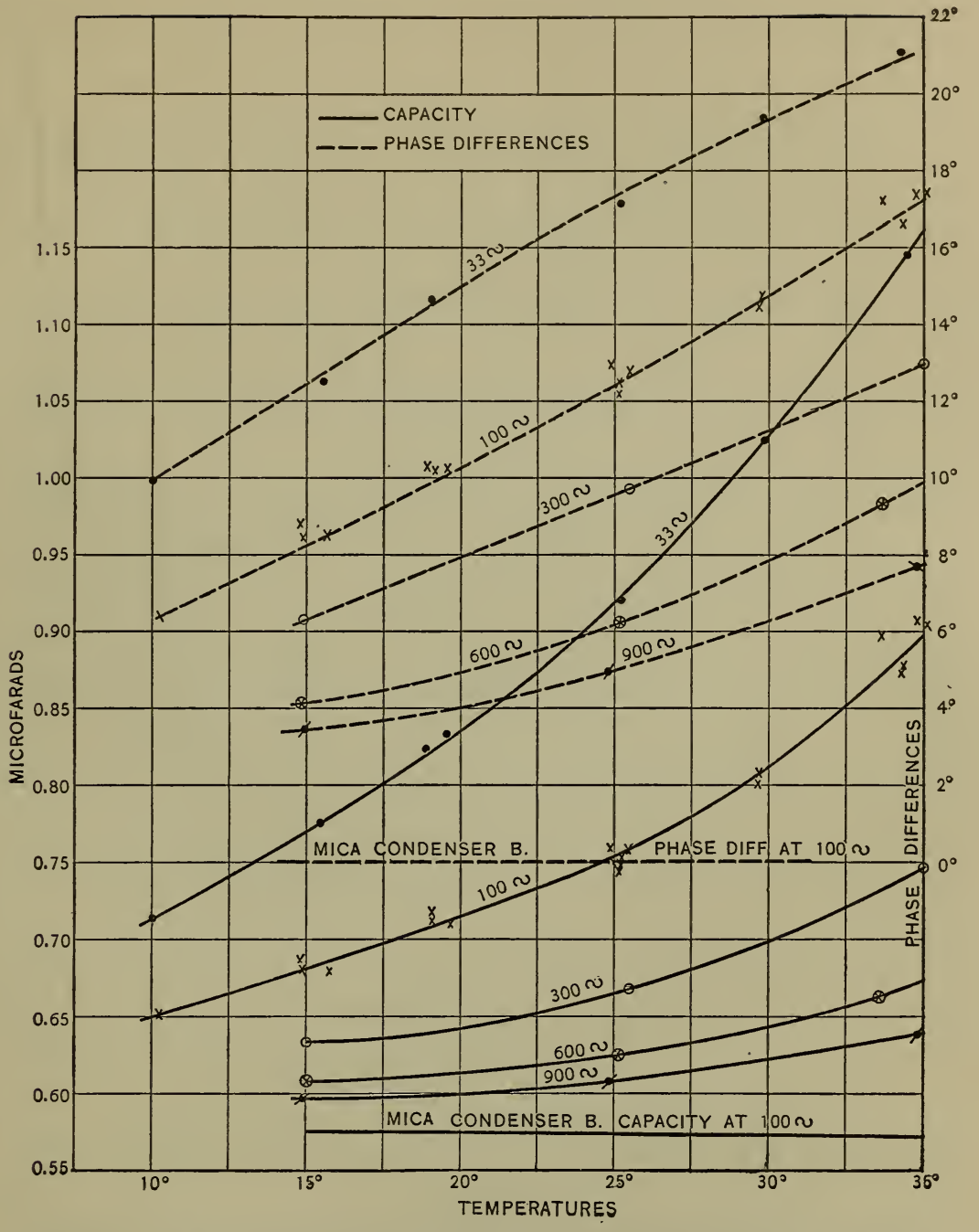

Fig. 29.—*Curves of Capacity and Phase Difference of* t_3 *as Functions of the Temperature*

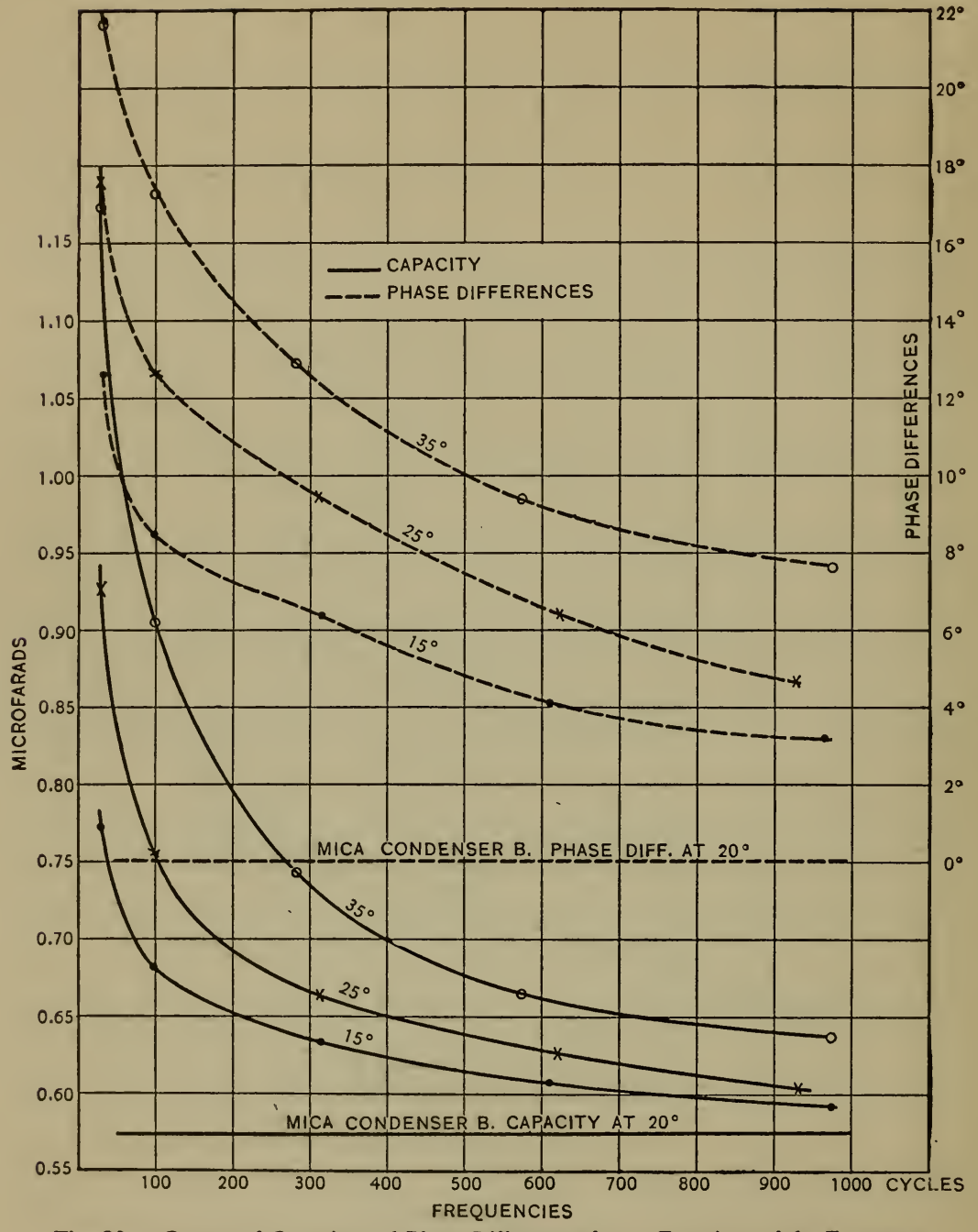

Fig. 30.—*Curves of Capacity and Phase Difference of* t_3 *as Functions of the Frequency*